PRESIDENCY UNIVERSITY, KOLKATA

M. Sc. Admission Test, 2013

for admission to the 1st Semester M. Sc. in Physics

Time 2 hrs. Full Marks 100

Each question carries 5 marks

1. Show that $\int_{-\infty}^{\infty} \frac{\sin x}{x} dx = \pi$.

2. A particle of mass m is free to move along the circumference of a vertical circle of radius a situated in the gravitational field of the earth. Obtain an expression for the Hamiltonian of the particle.

3. Two point masses m_1 and m_2 are connected by a massless inextensible string of length l. The string is passing through a hole in a smooth table such that m_1 is on the table surface and m_2 hangs suspended. Assume that m_2 undergoes only vertical motion while m_1 undergoes motion in the plane of the tables. Derive the equation(s) of motion and show that the angular momentum of the system is conserved.

4. Compute the flux of water (density = 1 ton/m^3) through the parabolic cylinder $\mathbf{R}$:$y = x^2$, $0 \leq x \leq 2$, $0 \leq z \leq 3$, if the velocity vector is $\mathbf{v} = (3z^2, 6, 6xz)$. The speed is measured in m/s.

5. A neutral π^0-meson decays into two high energetic photons. The rest mass of the π^0-meson is 135 MeV/c^2.
   (a) Find the energy of the photons if the π^0-meson decays at rest.
   (b) If the π^0-meson has total energy of 426 MeV in the laboratory system and decays in flight, what are the limits on the energy of the photon?

6. A rocket ship of proper length L leaves the earth vertically at a speed of $4c/5$. A light signal sent vertically reaches the tail of the rocket at $t=0$ according to both earth and rocket clocks. When does the signal reach the nose of the rocket according to (a) rocket clock (b) earth clock? Do not use Lorentz transformation.

7. Calculate the percentage of molecules of an ideal gas having free paths larger than twice the mean free path.

8. An electric current of 10 A is maintained for 1 s in a resistor of 25 Ω and at an initial temperature 27 °C. The resistor has mass 0.01 kg and C_p = 0.84 KJ/kgK. Find the entropy change of the resistor if (i) its temperature is maintained at 27 °C and (ii) it is thermally insulated.

9. Four perfect polarising plates are stacked so that the axis of each is turned 30° clockwise with respect to the preceding plate. How much of the intensity of an incident unpolarised beam of light is transmitted by the stack?

10. A parallel beam of light of wavelength 5460 Å is incident at an angle of 30° on a plane transmission grating having 6000 lines/cm rulings. Find the highest order spectrum that can be observed in that system.

11. A pure crystalline material with lattice constant 4 Å is having no impurities or dopants present in the structure. If the material illuminated by white light appears red in transmitted light, calculate the approximate band gap for this material.

12. Operator L is defined by LP = [H,P], where P is another operator and H is some fixed operator. Show that L is a linear operator.

13. (a) Which of the following functions can be chosen as a valid solution of Schrödinger equation? $e^{-r^2}, x^2, e^{ikx}, e^{-x}$. What type of solution (bound state or scattering state) do they represent?
(b) Is the operator d/dx Hermitian in the interval $(-\infty, \infty)$?

14. Test if the matrix $\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$ can be diagonalised, where a is a real number.

15. Obtain the electric field of a point dipole with dipole moment $\vec{p}$, if the potential due to it is $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}.\vec{r}}{r^3}$.

16. An ideal operational amplifier has the output connected to the inverting input terminal through a 500 Ω resistance. A 5 V rms ac signal is applied through a series resistance of 5 kΩ between the inverting input terminal and the ground. Calculate the output voltage and the current through the 500 Ω resistor.

17. What acceleration potential must be applied to electrons to cause electron diffraction on (220) planes of gold (*Au*) at 5 °C? Given: atomic volume of *Au* having FCC structure = 10.2×10^{-6} m^3/mole.

18. Consider a system of 3 noninteracting spin ½ particles in a magnetic field B. If the system has energy $-\mu B$, where $\mu$is the component of magnetic moment along the field direction, find the probability that the first spin will assume the value of $+\mu$ and determine the mean value of such a spin.

19. What is the kinetic energy of the α particle of the at-rest decay of $Po_{84}^{212} \rightarrow Pb_{82}^{208} + \alpha$? Given $(\Delta M)c^2$ = 8.95 MeV, where ΔM is the mass difference between that of initial and final particles.

20. State, with suitable justification, whether the following two elementary particle decay and production processes are allowed or forbidden.
(i) $n \rightarrow e^+ + e^-$ (ii) $\pi^- + p \rightarrow \Sigma^- + K^+$