Presidency University,
Kolkata
Department of Mathematics

Syllabus for B. Sc. Mathematics (Major)

PRESIDENCY UNIVERSITY, KOLKATA

SYLLABUS FOR THREE-YEAR B.Sc. **MATHEMATICS HONOURS** COURSE

Modules of the Syllabus

Semester	Paper	Module	Topics	Marks	
1	1	M101	Algebra I (Basic Algebra)	50	First Year (200 Marks)
	2	M191	Analysis I	50	
2	3	M201	Algebra II (Groups & Rings)	50	
	4	M291	Analysis II	50	
3	5	M301	Algebra III (Fields and Vector Spaces)	50	Second Year (300 Marks)
	6	M302	Metric Topology	50	
	7	M391	Computer Programming and data Structures	50	
4	8	M401	Analysis III	50	
	9	M402	Geometry I	50	
	10	M491	Complex Analysis	50	
5	11	M501	Algebra IV (Matrix Algebra I)	50	Third Year (500 Marks)
	12	M502	Discrete Mathematics and Number Theory	50	
	13	M503	Probability Theory	50	
	14	M591	Geometry II	50	
	15	M592	Mathematical Methods	50	
6	16	M601	Analysis IV (Theory of ODEs)	50	
	17	M602	Numerical Analysis	50	
	18	M603	Optimization Techniques	50	
	19	M691	Classical Mechanics	50	
	20	M692	Elective	50	

Each module of 50 marks will be evaluated as follows: 35 marks by end semester examination and 15 marks by internal assesment (class test / assignment etc).

Detailed Syllabus

M101 (50 Marks)

Algebra I (Basic Algebra)

Intuitive set theory; partial order; equivalence relations and partitions; Countable and uncountable sets; Zorn's lemma and the well ordering principle; principle of (transfinite) induction.

Quick review of algebra of complex numbers including De-Moivre's Theorem. Polynomials with complex co-efficients: Fundamental theorem of Algebra (statement only). Polynomials with real co-efficients: Nature of roots of an equation (surd or complex roots occur in pairs). Statements of Descartes rule of signs and of Sturm's theorem and their applications. Multiple roots. Relation between roots and coefficients. Symmetric functions of roots.

A review of modular arithmetic.

Matrices and determinants: basic properties, row/column operations; adjoint and inverse of a matrix. Groups, subgroups, normal subgroups, quotient groups; homomorphisms, permutation groups.

M191 (50 Marks)

Analysis I

Real numbers, functions, sequences, continued fractions, limits, limsup, liminf, series, tests for convergence, absolute convergence, rearrangement of terms, Cauchy product. Continuous functions of one real variable. Differentiation. Chain rule. Rolles theorem. Mean value theorem. Higher order derivatives. Leibnitz formula. Taylor series expansion. LHospitals rule. Maxima and minima of functions. Functions of bounded variations.

M201 (50 Marks)

Algebra II (Groups and Rings)

Isomorphism theorems; group actions; orbit-stabiliser theorem; conjugacy; Sylow's theorem; simple group, direct product; structure of finite abelian groups (statement only).

Definitions of rings and fields; ideals and quotient ring; ring homomorphisms and isomorphism theorems for rings; prime and maximal ideals; integral domains; characteristic; field of fractions; Euclidean domains; unique factorisation domains; principle ideal domains; polynomial rings.

M291 (50 Marks)

Analyssis II

Riemann integration. Fundamental theorem of Calculus. Computation of definite integrals. Improper integrals, Beta & Gamma functions. Sequences and series of functions. Point wise and uniform convergence. Term-by-term differentiation and integration. Power series, Weierstrass approximation theorem.

M301 (50 Marks)

Algebra III (Fields and Vector Spaces)

Field extensions, splitting fields, algebraic closure, separability, normal extensions.

Finite fields: structure of finite fields; existence and uniqueness theorems; primitive elements; minimal polynomials of elements; irreducible and primitive polynomials.

Vector spaces: subspaces and quotient spaces; homomorphism and isomorphism theorems; bases and dimension.

M302 (50 Marks)

Metric Topology

Elements of metric space theory sequences and Cauchy sequences and the notion of completeness, construction of real numbers, elementary topological notions for metric spaces i.e. open sets, closed sets, compact sets, connectedness, continuous and uniformly continuous functions on a metric space. The Bolzano - Weierstrass theorem, supremum and infimum on compact sets. Separability. Completeness. The Baire Category Theorem. $\mathbb{R}^n$ as a metric space.

Banach contraction principle. $C(X)$ as a metric space.. Arzela-Ascoli Theorem (statement only: proof can be given if time permits). Stone-Weierstrass Theorem (statement only).

M391 (50 Marks)

Computer Programming and Data Structures

Introduction to programming in the C language: arrays, pointers, functions, recursive programming and linked lists, file handling; notion of algorithms and their complexity, order notation; lists, stacks, queues and trees; searching and sorting algorithms; string algorithms; object-oriented programming and introduction to C++.

M401 (50 Marks)

Analysis III

Functions of several variables. Continuity. Partial derivatives. Differentiability. Taylors theorem. Multiple integrals. Repeated integrals. The Jacobian theorem. Line, surface and volume integrals. Green's Theorem. Statements of Inverse and Implicit Function Theorems. Maxima and minima. Lagrange multiplier.

M402 (50 Marks)

Geometry I

Quick review of two-dimensional coordinate geometry, specially conics and system of circle.

Rectangular Cartesian co-ordinates, cylindrical, polar and spherical polar co-ordinates in 3 dimensions. Projection of a vector on a co-ordinate axis. Inclination of a vector with an axis. Direction cosines of a vector. Distance between two points. Division of a directed line segment in a given ratio.

Planes: Equation of a plane, signed distance of a point from a plane. Equation of a plane passing through the intersection of two planes. Angle between two intersecting planes. Bisectors of angles between two intersecting planes. Parallelism and perpendicularity of two planes. Lines in space: Equations of a line. Rays or half lines. Direction cosines of a ray. Angle between two rays. Distance of a point from a line. Condition of coplanarity of two lines. Skew-lines. Shortest distance.

Curves in two and three dimensions. Parametrized curves, re-parametrization. Regular and singular points. Curvature and torsion for space curves. Existence theorem for space curves. Serret-Frenet formula for space curves.

M491 (50 Marks)

Complex Analysis

Holomorphic functions and the Cauchy-Riemann equations, Power series, Functions defined by power series as holomorphic functions, Complex line integrals and Cauchys theorem, Cauchys integral formula. Representations of holomorphic functions in terms of power series. Liouvilles theorem, Zeros of analytic functions, The fundamental theorem of algebra. Poles, Singularity, Meromorphic functions and Laurent series. The maximum modulus principle, Schwarzs lemma, The argument principle, The open mapping property of holomorphic functions, Conformality, Mobius transformations and The Cross Ratio. The calculus of residues and evaluation of integrals using contour integration. Complex Logarithm.

M501 (50 Marks)

Algebra IV

Linear transformations and their matrices; row and column spaces of a matrix; rank and nullity of a matrix; rank factorisation; system of linear equations.

Eigenvalues and eigenvectors; algebraic and geometric multiplicities; characterization of diagonalizable matrices; characteristic and minimal polynomial; Caley-Hamilton Theorem.

Bilinear forms, inner products, Gram-Scmidt orthogonalization process; orthogonal spaces; direct sum : quadratic forms, positive and negative definite matrices; extrema of positive definite quadratic forms.

Definition of unitary, hermitian, normal, real symmetric and orthogonal matrices. Statement of spectral theorems for real symmetric matrices.

M502 (50 Marks)

Discrete Mathematics and Number Theory

(a) Combinatorics:
Basic counting principles; multinomial theorem; principle of inclusion exclusion; Recurrence

relations and classification, summation method, extension to asymptotic solutions from solutions for subsequences; Linear homogeneous relations, characteristic root method, general solution for distinct and repeated roots, non-homogeneous relations and examples; generating functions and their application to linear homogeneous recurrence relations.

(b) Number Theory:
Diophantine equations, divisibility, primes and unique factorisation; GCD and Euclidean algorithm and its extension for computing multiplicative inverses; congruences and complete residue systems; Fermat, Euler, Wilson theorems; linear congruences and the Chinese remainder theorem; Hensel's Lemma; primitive roots and index calculus; quadratic residues and the quadratic reciprocity theorem, Jacobi symbol, square roots.

M503 (50 Marks)

Probability Theory

Classical Theory & Its Limitations: Random experiment and events, event space; classical definition of probability and its drawbacks, statistical regularity, frequency definition and its drawbacks.

Probability Axioms: Basics from measure theory, probability measure, probability space, continuity theorem in probability, exclusion-inclusion formula, conditional probability & Baye's rule, Boole's inequality, Independence of events, Bernoulii trials and binomial law, Poisson trials, Probability on finite sample spaces, Geometric probability.

Random variables and Their Probability Distributions: Random variables, Probability distribution of a random variable, discrete and continuous random variable, some discrete and continuous distributions on IR: Bernoulli, Binomial and Poisson; uniform, normal, Gamma, Cauchy and χ^2-distribution; functions of a random variable and their probability distribution.

Characteristics and Generating Functions: Expectation, moments, measures of central tendency, measures of dispersion, measures of skewness and kurtois, Markov & Chebycheff's inequality, probability generating function, moment generating function, characteristic function.

Probability Distributions on $\mathbb{R}^n$: Random vectors, Probability distribution of a random vector, functions of random vectors and their probability distributions, moments, generating functions, correlation coefficient, conditional expectation, the principle of least squares, regression.

Convergence and Limit Theorems: sequence of random variables, convergence in distribution, convergence in probability, almost sure convergence, convergence in rth mean, weak and strong law of large numbers, Borel-Cantelli lemma, limiting characteristic functions, central limit theorem (statement only).

M591 (50 Marks)

Geometry II

Review of calculus of several variables. Proof of Inverse Function Theorem and Implicit Function Theorem. Differential forms in $\mathbb{R}^n$. Exterior derivative of forms. Closed and exact forms. Surfaces in $\mathbb{R}^3$, tangent plane and normal. Orientation of a surface. Forms on surfaces. Integration on surfaces. Stokes formula. Generalization to regular level-surfaces in $\mathbb{R}^n$.

Definition of differentiable manifolds. Surfaces as two dimensional manifolds.

If time permits, following topics can be touched upon:

Tangent space and derivative of maps between manifolds. First fundamental form. Second fundamental form and the Gauss map. Mean curvature and scalar curvature. Statement of the Gauss- Bonnet theorem.

M592 (50 Marks)

Mathematical Methods

First Order ODE: Separation of variables, integrating factor, Bernoulli equation, Clairaut's form, singular solution.

Higher Order Linear ODE with Constant Coefficients: Complementary function, particular integral, method of undetermined coefficient, method of variation of parameters, use of operator D.

Second Order Linear ODE with Variable Coefficients: Method of reduction of order, method of variation of parameter.

Power Series Solutions: Series solution of first order ode, second order linear ode, ordinary and regular singular points, power series solution of second order linear ode around regular singular points, Hypergeometric equation and hypergeometric functions.

Laplace Transformations: Introduction to integral transformations, Laplace transforms properties of Laplace transforms, convolutions, inverse formula for Laplace transforms, solving differential equations using Laplace transforms.

Partial Differential Equation(PDE): An introduction and formation of PDE. Method of characteristics to solve first order PDEs.

M601 (50 Marks)

Analysis IV (Theory of ODEs)

Fundamental theorem for existence and uniqueness, Gronwall's inequality and more comparison theorems, Dependence on initial conditions, maximal interval of existence, Global existence of solutions, vector fields and flows, Topological conjugacy and equivalence.

Linear Systems in $\mathbb{R}^n$: The matrix exponential, linear first order autonomous systems, Jordan Canonical forms, invariant subspaces, stability theory, fundamental matrix solution, Non homogeneous linear systems, Periodic linear systems and Floquet theory.

Nonlinear systems : Critical points, Stability and Lyapunov's method, linearization and Hartman Grobman Theorem (statement only), invariant manifolds, limit sets, periodic orbits and limit cycles, stability of periodic orbits: Poincaré map and Floquet theory.

The Phase Plane: Nonhyperbolic critical points and its classifications, imaginary eigenvalues and topological centers, separatrix cycles, Poincaré Benedixson theorem, Benedixson's criteria.

M602 (50 Marks)

Numerical Analysis

Numerical solution of equations: Determination of real roots: Bisection method. Regula-Falsi method and modification, Secant method, Newton-Raphson methods, their geometrical significance.

Fixed point iteration method. Complex roots: Mullers method.

Errors in Numerical computation: Approximation and errors in numerical computation.

Interpolation: Problems of interpolation, Weierstrass approximation theorem (only statement). Polynomial interpolation. Equispaced arguments. Difference table. Deduction of Newtons forward and backward interpolation formulae. Statements of Stirlings and Bessels interpolation formulae. Error terms. General interpolation formulae: Deduction of Lagranges interpolation formula. Divided difference. Newtons General Interpolation formula (only statement). Inverse interpolation.

Interpolation formulae using the values of both $f(x)$ and its derivative $f(x)$: Idea of Hermite interpolation formula (only the basic concepts).

Algorithms and Programming for numerical methods in C (or C++).

M603 (50 Marks)

Optimization Techniques

Linear programming: basic formulation; geometric interpretation and convex polytope; simplex algorithm, Bland's rule, duality theory, complexity of simplex method; polynomial time algorithms ellipsoidal and Karmarkar's methods ".

Network and graph problems: minimum spanning trees, shortest path, flows in networks, perfect matching problem; Gale-Shapley algorithm for stable marriage.

Brief introduction to integer programming and non-linear programming problems.

M691 (50 Marks)

Classical Mechanics

1. Review of Newtonian Mechanics: Newton's equation of motion, simple illustrations: motion in central field, motion of n-particles.

2. Lagrangian Mechanics: Constraints, degrees of freedom, D' Alembert's principle, generalized coordinates, derivation of Lagrange's equation for conservative holonomic system from D' Alembert's principle and from variational principle, solution of problems by Lagrange's equation.

3. Hamiltonian Mechanics: The Hamiltonian, Hamilton's canonical equations and variational principle, Noether's theorem, conservative principles, Poisson brackets and the canonical transformations, Hamilton-Jacobi equation, action-angle variable, Liouville's theorem.

M692 (50 Marks)

Elective: Linear and Matrix Algebra II

Companion form; rational form and Jordan form of a matrix (without proof); Lower and upper bounds for rank of product of two matrices.

Elementary operations and elementary matrices, Echelon form, Normal form, Hermite canonical form and their use (sweep-out method) in solving linear equations and in finding inverse. LDU-decomposition.

Formulae of determinant and inverse of a partitioned matrix; idempotent matrices; left inverse and right inverse of full-rank rectangular matrices; generalized inverse.

Proof of spectral theorem for complex hermitian and real symmetric matrices; singular value decomposition; polar decomposition; simultaneous diagonalization of commuting hermitian/ real symmetric matrices.

Elective: Discrete Mathematics and Number Theory II

a) Number Theory:
Public key cryptography: primality testing using Rabin-Miller algorithm, idea of hardness of factoring and discrete logarithm; basics of Diffie-Hellman Key Agreement and RSA cryptosystem and digital signatures.

Cyclotomic polynomials, arithmetic functions, Mobius inversion formula, zeta functions; continued fractions, periodic continued fractions, quadratic irrationalities; Brahmagupta-Pell Equation; four squares theorem; Fermat descent.

(b) Graph Theory:
Graphs and digraphs, complement, isomorphism, connectedness and reachability, adjacency matrix, Eulerian paths and circuits in graphs and digraphs, Hamiltonian paths and circuits in graphs and tournaments, trees; rooted trees and binary trees, planar graphs, Euler's formula, statement of Kuratowski's theorem, dual of a planer graph, independence number and clique number, chromatic number, statement of Four-color theorem, dominating sets and covering sets.

Elective: Theory of Computing

Automata and Languages: Finite automata, regular languages, regular expressions, closure properties, equivalence of deterministic and non-deterministic finite automata, pumping lemma, minimisation of finite automata.

Context-free languages: context-free grammars, closure properties, pumping lemma for CFL, push down automata.

Computability: Turing machines and computable functions, universality, halting problem, recursive and recursively enumerable sets.

Complexity: Time complexity of deterministic and nondeterministic Turing machines; basic idea of the classes P and NP; notion of NP-completeness and brief idea of reducibility among NP-complete problems.

Elective: Analysis

Fourier series, Fourier Transform, Laplace Transform. Solution to Differential Equations using Laplace Transforms.

Elective: Game Theory

Decision making and conflict; two-person, zero-sum game; pure and mixed strategy; saddle point and its existence.

Optimal strategy and value of the game; maximum and minimax solution. Games in normal form: notions of domination; rationalisable strategies. Nash equilibrium: existence, properties and applications. Games in extensive form: credibility and sub-game perfect Nash equilibrium. Introduction to bargaining and repeated games.

Cooperative game theory: concept of core and nucleolus; Bondareva-Shapely theorem; relation to linear programming (if time permits); power indices; stable marriage and the Gale-Shapley algorithm.

Elective: Statistics

Descriptive statistics: Population and sample; frequency distribution and applications; cumulative graph and histogram. Measures of central tendency: mean, median, mode, quantiles. Measures of dispersion: mean deviation; root mean square; variance and standard deviation; moments and moment generating functions; characteristic function; skewness and kurtosis. Sample characteristics; sampling distribution; χ^2, t and F distributions.

Sufficient statistic, likelihood function, Fisher-Neyman factorisation theorem, ancillarity and Basu's theorem. Maximum likelihood estimation; unbiased estimator; Cramer-Rao theorem; uniformly minimum variance unbiased estimator; Hypothesis testing: Type-I, Type-II errors, Neyman-Pearson theorem; likelihood ratio testing; interval estimation. Bivariate samples, sample correlation co-efficient, least square curve fitting, regression lines.

Introduction to R to be provided at the beginning of the course and relevant programming exercises to be given as the course proceeds.

References

Reference Texts for Algebra I-IV and Matrix Algebra:

1. M. Artin: Algebra.
2. S. D. Dummit and M. R. Foote: Abstract Algebra.
3. I. N. Herstein: Topics in Algebra.
4. C. R. Rao: Linear Statistical Inference and Its Applications.
5. A. Ramachandra Rao and P. Bhimasankaram: Linear Algebra.
6. K. Hoffman and R. Kunze: Linear Algebra.
7. F. E. Hohn: Elementary Matrix Algebra.
8. P. R. Halmos: Finite Dimensional Vector Spaces.
9. R. B. Ash: Abstract Algebra: The Basic Graduate Year. Free download from http://www.math.uiuc.edu/ r-ash/Algebra.html.

Reference Texts for Analysis I-III and Metric Topology:

1. W. Rudin: Principles of Mathematical Analysis.
2. Tom Apostol: Mathematical Analysis.
3. Tom Apostol: Calculus I and II.
4. Terence Tao : Analysis I.
5. W. Rudin: Real and Complex Analysis.

Reference Texts for Computer Programming and Data Structures:

1. Brian Kernighan and Dennis Ritchie: The C Programming Language.
2. Ellis Horowitz and Sartaj Sahani: Fundamentals of Data Structures.

Reference Texts for Geometry I and II:

1. M.P. do Carmo: Differential Geometry of Curves and Surfaces.
2. J. A. Thorpe: Eelementary Topics in Differential Geometry.
3. Spivak: Calculus on manifolds.

Reference Texts for Complex Analysis:

1. Elias M. Stein, Rami Shakarchi: Complex Analysis.
2. Lars Ahlfors: Complex Analysis.
3. T. W. Gamelin: Complex Analysis.
4. J.B.Conway: Functions of One Complex Variable.

Reference Texts for Discrete Mathematics and Number Theory:
I. Number Theory:

1. Ivan Niven, Herbert S. Zuckerman and Hugh L. Montgomery. An Introduction to the Theory of Numbers.
2. E. M. Wright and G. H. Hardy. An Introduction to the Theory of Numbers.
3. Open Courseware from MIT. http://ocw.mit.edu/courses/mathematics/18-781-theory-of-numbers-spring- 2012/lecture-notes/.

II.Combinatorics and Graph Theory:

1. Fred S. Roberts. Applied Combinatorics.
2. Frank Harary. Graph Theory.
3. Douglas West. Introduction to Graph Theory.

Reference Texts for Probability Theory:

1. William Feller: An Introduction to Probability Theory and its Applications.
2. David Stirzaker and Geoffrey Grimmett: Probability and Random Processes.
3. V. K. Rohatgi and A. K. Md. Ehsanes Saleh: An Introduction to Probability and tatistics.
4. Rick Durett: Probability Theory & Examples.

Reference Texts for Mathematical Methods:

1. G.F. Simmons: Differential Equation with Applications and Historical Notes.
2. A.C. King, J. Billingham and S.R. Otto: Differential Equations: Linear, Nonlinear, Ordinary, Partial.

Reference Texts for Analysis IV (Theory of ODEs):

1. C. Chicone: Ordinary differential Equations with applications.
2. L.D. Perko: Differential Equations and Dynamical Systems.

3. E. A. Coddington and N. Levinson: Theory of Ordinary Differential Equations.

Reference Texts for Numerical Analysis:

1. Kendall Atkinson: An Introduction to Numerical Analysis.
2. Brian Kernighan and Dennis Ritchie: The C Programming Language.
3. W.H. Press, S.A. Teukolsky, W.T. Vettering, B.P. Flannery: Numerical Recipes in C.

Reference Texts for Optimization Techniques:

1. C. H. Papadimitriou and K. Steiglitz: Combinational Optimization.
2. Robert J. Vanderbei: Linear Programming.
3. David Luenberger: Linear and nonlinear programming.

Reference Texts for Classical Mechanics:

1. Rana & Joag : Classical Mechanics.
2. Herbert Goldstein: Classical Mechanics.
3. A. K. Raychaudhuri: Classical Mechanics: A Course of Lectures.

Reference Texts for Theory of Computing:

1. J. E. Hopcroft and J. D. Ullman: Introduction to Automata Theory, Languages and Computation.
2. H. R. Lewis and C. H. Papadimitriou: Elements of The Theory of Computation.
3. M. Sipser: Introduction to The Theory of Computation.

Reference Texts for Game Theory:

1. M. Osborne and A. Rubinstein: A Course in Game Theory.
2. R. Myerson: Game Theory.
3. D. Fudenberg and J. Tirole: Game Theory.
4. S.R. Chakravarty, M. Mitra and P. Sarkar: A Course in Cooperative Game Theory.