# PRESIDENCY UNIVERSITY

**Syllabus for two-year MATHEMATICS M.Sc. Course**

## Outline of the Syllabus

<table>
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<th>Module</th>
<th>Paper</th>
<th>Topic</th>
<th>Marks</th>
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<tr>
<td><strong>Semester-I (250 Marks)</strong></td>
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<tr>
<td>M411</td>
<td>1</td>
<td>Algebra-I (Group, Ring, Field, Galois Theory)</td>
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<tr>
<td>M412</td>
<td>2</td>
<td>Real Analysis-I (Measure and Integration, Summability)</td>
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<tr>
<td>M413</td>
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<td>Algebra-II (Modules, Vector Spaces)</td>
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<td>Analytical Mechanics</td>
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<td>Real Analysis-II (Metric Spaces)</td>
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<td>M422</td>
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<td>Differential Equations and Special Functions</td>
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<td>Differential Geometry</td>
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<td>Numerical Analysis</td>
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<td>M514</td>
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<td>Computer Programming (Practical)</td>
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<td>M515</td>
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<td>Mathematical Methods</td>
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<td><strong>Semester-IV (250 Marks)</strong></td>
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<td>Project</td>
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Detailed Syllabus

M411 (50 Marks)
(Algebra I (Group, Ring, Field, Galois Theory))

1. Group:
   (a) Group Actions: Group Actions and Permutation. Representations. Groups Acting on Themselves by Left Multiplication - Cayley’s Theorem.
   (b) Structure of groups: Free Abelian Groups. Finitely Generated Abelian Groups.
   (c) Composition Series. Nilpotent Groups and Solvable Groups.

2. Ring, Field:
   (b) Field extensions. Algebraic Extensions. Splitting Fields, Algebraic Closure and Normality.

3. Galois theory:
   (b) Solvability of equations by radicals. Finite Fields.
   (c) Transcendental Extensions, Inseparable Extensions, Infinite Galois Groups.

M412 (50 Marks)
(Real Analysis - I (Measure and Integration, Summability etc))

1. Functions from $\mathbb{R}^m$ to $\mathbb{R}^n$, their continuity, differentiability, differential, chain rule, inverse function theorem, Implicit function theorem.

2. Measure: $\sigma$-algebra, Lebesgue outer and inner measure, measurable set, measurable function, Borel set, Borel function.


4. Square summable functions, introduction to $l_p$ and $L_p$-spaces.

M413 (50 Marks)
(Algebra - II (Modules, Vector Spaces))


M414(50 Marks)
(Complex Analysis)

1. Complex numbers as ordered pair, Geometric representation of Complex numbers. Topology of \( \mathbb{C} \). Stereographic projection, point at infinity and the extended complex plane \( \mathbb{C}_\infty \).


5. Analytic continuation.

M415(50 Marks)
(Analytical Mechanics)

1. Mechanics of a system of particles: Constraints; Generalized coordinates; D’Alembert’s principle; Generalized forces; Lagrangian; Lagrange’s equations of motion; cyclic coordinates; Equation of energy; Rotating axes of coordinates; Coriolis’s force.

2. Hamiltonian: Hamilton’s equation of motion; Hamilton’s principle; Principle of least action; Fermat’s principle; Symmetry properties and conservation laws; Noether’s theorem; Holonomic and Non-holonomic systems.

3. Canonical Mechanics: Canonically conjugate coordinates and momenta; Legendre transformation; Canonical transformation; Generating function; Poisson Brackets; Hamilton-Jacobi’s equation; Poincaré-Cartan integral invariant; Liouville’s theorem.

4. Rigid Dynamics:
   (a) 3D Kinematics of a rigid body; Euler angles; Cayley-Klein parameters; Finite and infinitesimal rotations; Inertia tensor; Principal axis transformation; Euler’s dynamical equations; Motion of a free top about a fixed point–precession and nutation.
   (b) Small oscillations about equilibrium; Normal coordinates; Oscillations under constraints; Stationary character of a normal mode.

M421(50 Marks)
(Real Analysis II (Metric Spaces))

1. Open sets, local base, second countable and separable metric spaces – their equivalence, metric subspaces.

2. Sequences: convergence and clustering, subsequences, Cauchy sequence, completeness, Baire’s category theorem, completeness of \( C[a, b] \). Completion of metric spaces, \( \mathbb{R} \) as completion of \( \mathbb{Q} \).

3. Continuity: definition, equivalent conditions, composition of continuous functions,

5. Connectedness: definition and elementary results, connectedness in $\mathbb{R}$, local connectedness, structure of open sets in locally connected second countable metric spaces with special reference to $\mathbb{R}$ and $\mathbb{R}^n$.


**M422(50 Marks)**

(Differential Equations and Special Functions)

1. *Ordinary Differential Equations*
   
   
   (b) Linear inhomogeneous differential equation: Solution by variation of parameters.
   
   

2. *Partial Differential Equations*
   
   

**M423(50 Marks)**

(General Topology)

1. Basic concepts: Open and closed sets, bases and subbases for open sets and for closed sets, iff conditions, closure operator, neighbourhood, closure and interior — related concepts, comparison of topologies, subspaces, sequence and their convergence.

2. Continuity of functions, quotient spaces, homeomorphism, embedding.

3. Separation axioms, examples of different spaces $T_0, T_1, T_2, T_3$. Tychonoff, and $T_4$, related theorems, Urysohn’s Lemma, Tietz’s extension theorem.

5. Inadequacy of sequential convergence: nets filters and convergence.

6. Connectedness, local connectedness and related results. Structure of open sets in locally connected separable topological spaces with special reference to metric spaces, $\mathbb{R}$ and $\mathbb{R}^n$.


M424(50 Marks)
(Mathematical Logic)


4. Hilbert type system, Natural Deduction system and Gentzen sequent calculus for first order predicate logic - their interrelation.

M425(50 Marks)
(Continuum Mechanics)

1. General Tensors

2. Geometrical Foundations: Lagrangian and Eulerian systems

3. Analysis of Deformation: Strain tensor

4. Analysis of Stress: Stress tensor

5. Balance Laws: Mass, momentum and energy; Equations of motion; Energy equation

6. Nonlinear Constitutive Equations

7. Classification of materials: Finite and infinitesimal elasticity; Viscous Newtonian fluids
M511 (50 Marks)
(Functional Analysis)

0. Linear space. Functionals.


M512 (50 Marks)
(Differential Geometry)

1. Geometry of curves in \( \mathbb{R}^2 \) and \( \mathbb{R}^3 \).
   (a) Parametrized curves. Regular curves, arc length.
   (b) The vector product in \( \mathbb{R}^3 \).
   (c) Local theory of curves: Curvature and torsion of a curve. Frenet-Serret formulas for curves.
   (d) Isoperimetric inequality.

2. Geometry of Surfaces in \( \mathbb{R}^3 \).
   (a) Regular surfaces. Inverse image of a regular value of a function.
   (b) Examples of regular surfaces: Sphere, torus, cylinder, surfaces of revolution, quadric surfaces.
   (c) Differentiable functions on a surface.
   (d) Tangent plane. The differential of a map.
   (e) The first fundamental form. Area.
   (f) Vector fields on a surface. Integral curves of a vector field.
   (g) Parallel vector fields on a surface along a parametrized curve.
   (h) Geodesics.
   (i) The definition of Gauss map and its fundamental properties.
   (j) The second fundamental form.
   (k) Normal curvature. Gaussian curvature. Theorem of Meisner.
   (l) Gauss map in local coordinates.
   (m) Geometric interpretation of Gaussian curvature
   (n) Definition of isometry and local isometry.
   (o) Gauss’s theorem Egregium
M513 (50 Marks)
(Numerical Analysis)


2. Eigenvalues and Eigenvectors of Real Matrix: Power method for extreme eigenvalues and related eigenvectors, Jacobi’s method for symmetric matrix (algorithm only), Given’s method and Householder’s reflections; QR method.

3. Solution of Non-linear Equations: Modified Newton-Raphson method; Aitken’s $\delta^2$-method and Steffensen’s iteration; Bairstow’s method.


5. Polynomial Interpolation: Runge’s phenomena, piecewise polynomial interpolation; Cubic spline interpolation.

6. Approximation of Functions: Least squares polynomial approximation, Approximation with orthogonal polynomials, Chebyshev polynomials economization, Harmonic analysis.

7. Numerical Integration: Problem of approximate quadrature, Trapezoidal and Simpson’s rule with error formula; Newton-Cotes formulae, Gauss-Legendre and Gauss-Chebyshev quadratures, Euler-Maclaurin summation formula, Richardson extrapolation, Romberg integration, Simpson’s adaptive quadrature, Double integrals and Cubature formula of Simpson Type, Improper integrals.

8. Numerical Solution of Initial Value Problems for ODE:
   - Two-point Boundary Value Problems for ODE: Finite difference methods, Shooting method.


M514 (50 Marks)
(Computer Programming (Practical))

1. Programming in C++
   (a) Character set, Keywords, Datatypes (character, integer, floating point, etc.), Constants, Variables, Operators (arithmetic, assignment, relational logical increment, etc.), Expressions, Data input and output (The functions like printf, scanf, etc.), Header files
   (b) Branching and Looping (The statements like if-else, while, do-while, for, switch, break, continue, etc.)
   (c) User-defined functions
   (d) Arrays, Strings, Pointers, Structures and Unions

2. MATLAB
(a) Constants, Variables, Operators, Functions, Relational expressions, Data input and output, Matrices and Matrix operations, In-built functions

(b) Looping, User-defined functions

(c) Handling System of Linear Equations and Eigensystems

(d) Solution of non-linear equations

(e) Differentiation and Integration

(f) Solution of ODEs

(g) Graphics (2D & 3D)

(h) Curve-fitting

(i) Handling Optimization problems

(j) Applications of Symbolic Toolboxes

M515(50 Marks)
(Mathematical Methods)

1. Integral Transforms

   (a) Laplace transform: Existence, Uniqueness, Inversion, Convolution theorem. Convergence, Shifting theorems, Applications to ODE.

   (b) Fourier transform: Existence, Uniqueness, Inversion, Applications to ODE & PDE.


4. Wavelets & wavepacket techniques

M521(100 Marks)
(Advanced Mathematical Logic)

Advanced Mathematical Logic - I (50 marks)

1. Modal logic: Propositional logic systems K, T, S4, B and S5. Soundness and completeness theorems with respect to Kripke models and algebraic models. Application in knowledge engineering.

2. Axiomatic set theory: ZFC

Advanced Mathematical Logic - II (50 marks)

1. Many-valued logic: General introduction to many-valued logics — Gödel logic, Lukasiewicz logic, Kleene logic and Post logic.


1. Basic Concepts: Dynamical models (deterministic and stochastic), Discrete-time and Continuous-time dynamical models.

2. Single-Species Populations: Discrete-time models (logistic difference equation, Beverton-Holt, and other difference equation models), Allee effect, Equilibrium analysis, Period doubling, bifurcation and chaos, Liapunov exponent, Cobwebs and bifurcation diagrams, Rabbit problem and Fibonacci sequence.


5. Numerical Simulation: Illustration of analytical results using MATLAB.

1. Exploited Populations: Harvest models, Pontryagin’s maximum principle and optimal harvest policies.


5. Curve Fitting and Biological Modeling: Least square curve fitting, Applications to various biological data.

6. Numerical Simulation: Illustration of analytical results using MATLAB.

1. Fundamental theory: Initial value problems (IVPs), Local existence and uniqueness of solutions of IVP, Continuation of solutions, Continuous dependence properties.

2. Linear systems: Fundamental matrices, Linear systems with constant coefficients, Two-dimensional linear autonomous systems, Linear systems with periodic coefficients, Adjoint systems.

(Advanced Differential Equations-II (50 Marks))

1. Lyapunov functions
3. Index theory in the plane, Brouwer’s degree in $\mathbb{R}^n$.

M524(100 Marks)
(Theory of Computation)

Theory of Computation - I (50 marks)

1. Formal languages and formal grammars.
   Chomsky-classification of languages.
   Algebra of languages.
2. Finite automata.
   Deterministic and non-deterministic finite automata.
   Regular expression.
   Interconnection between regular expression, regular language and finite automata.
   Pumping lemma for regular languages.
3. Context free languages and normal forms.
   Parse tree.
   Push down automata and context free languages.
   Pumping lemma for context free languages.

Theory of Computation - II (50 marks)

   Connection of Turing machine with formal grammars.
2. Elementary introduction to the theory of computational complexity.
   Notions of the class P and the class NP.

M525(100 Marks)
(Algebraic Topology)

Algebraic Topology-I (50 marks)

(a) Homotopy, homotopy class, null homotopy. Homotopy relative to a subset. Path homotopy. Fundamental group of a topological space. Homomorphism induced by a continuous map. Functorial properties of $\pi_1$.

(b) Simply connected spaces. Retraction, deformation retraction, strong deformation retraction, contractibility (Illustration with examples). Homotopy equivalence and homotopy inverse. Induced homomorphisms.

(c) Covering spaces $p: E \to X$. Examples of $\exp : \mathbb{R} \to S^1$. Finite coverings of $S^1$ by itself, namely $z \mapsto z^n$. Double covers of real projective spaces by spheres.


(e) No retraction theorem. Brouwer’s fixed point theorem. Fundamental theorem of algebra.

(f) Van Kampen’s theorem. Fundamental group of spheres of dimension greater than 1.

(g) Computation of Fundamental groups of the Torus and the real projective spaces. Example of a space with non-abelian fundamental group (e.g. figure eight).

(h) Jordan’s separation theorem. Invariance of domain.

(i) Covering transformations. Action of $\pi_1(X)$ on the covering space of $X$.

Algebraic Topology-II (50 marks)

Singular Homology Theory. Review of free abelian groups.

1. Singular simplex. Singular chain complex. Cycles and boundaries. Homology groups. Homomorphisms induced by continuous maps. Functorial properties of $H_n, n \geq 0$. Zero-th homology group of a path-connected space. Homology groups of a convex subspace of $\mathbb{R}^n$.

2. Eilenberg-Steenrod axioms. Verification of some of the axioms for singular homology theory. Dimension axiom and axiom on compact supports.

Chain homotopy. Homotopic maps to chain homotopies. (Verification of homotopy axiom may be omitted). Homotopy equivalence and the induced homomorphism at the homology level.

Hurewicz theorem connecting $H_1$ and $\pi_1$. Determination of $H_1(S^1)$.

Relative homology groups. Homology long exact sequence of a pair $(X, A)$.

Excision theorem. Mayer-Vietoris theorem.

3. Computation of homology groups of spheres.

No retraction theorem. Brouwer’s fixed point theorem.


5. Jordan’s separation theorem. Invariance of domain.
M526 (100 Marks)
(General Topology)

General Topology-I (50 marks)


General Topology-II (50 marks)

1. Rings of Continuous Functions.
2. $z$-ideals and $z$-filters, characterization of compactness.

M527 (100 Marks)
(Operations Research)

Operations Research-I (50 marks)

4. Application of LP theory to transshipment, scheduling and travelling salesman models.

Operations Research-II (50 marks)