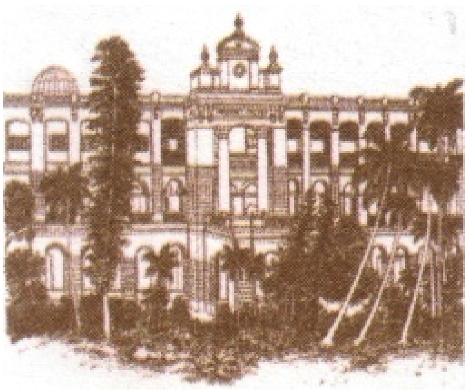
PRESIDENCY UNIVERSITY

DEPARTMENT OF MATHEMATICS

Syllabus for two-year M.Sc. Programme in Mathematics
(effective from the academic session 2021-22)

EXCELLENCE SINCE 1817

PRESIDENCY UNIVERSITY
KOLKATA



Department of Mathematics
(Faculty of Natural and Mathematical Sciences)
Presidency University
Hindoo College (1817-1855), Presidency College (1855-2010)
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West Bengal, India

Programme Outcomes

PO 1 Developing Analytical and Real-Life Skills: Students will be able to know the importance of mathematical modelling, simulation and computational methods to solve real world problems. They will be able to model physical, biological, environmental, statistical etc. problems using mathematical knowledge. They will be able to analyse and suggest acceptable real- life solutions using mathematical and data interpretation skills.

PO 2 Promoting Higher Education: Students completing this programme will be able to apply their knowledge in Mathematics to construct and develop logical arguments for the solution of complex mathematical problems, describe and formulate mathematical ideas from multiple perspectives. They will be able to explain and apply fundamental concepts of mathematics for solving advanced research problems.

PO 3 Enhancing Employability in Industry: Students will be able to use the knowledge acquired in related areas of computer science, statistics and Programming Languages to enhance their employability for government jobs, jobs in software engineering, data science, banking, insurance and investment sectors and in various other public and private enterprises.

PO 4 Inculcating Innovation and Creativity: Students will be able to undertake independent research initiatives in mathematics. They will be able to create and hypothesise mathematical results. Will be able to estimate and understand and analyse the limitations of a method and suggest appropriate remedies for tackling such problems.

Course Structure for two-year M.Sc. Programme in Mathematics
(with effect from the academic session 2021-22)
Semester-wise distribution of Courses

Semester	Paper Code	Name of the Courses	Page Number	Full Marks	Credit Point	Classes per week	Course Type †
I	MATH0701	Algebra - I	4	50	4	4 hr	T
	MATH0702	Topology - I	5	50	4	4 hr	T
	MATH0703	Ordinary Differential Equations	7	50	4	4 hr	T
	MATH0791	Classical Mechanics	9	50	4	4 hr	S
	MATH0792	Complex Analysis	11	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
II	MATH0801	Algebra - II	13	50	4	4 hr	T
	MATH0802	Geometry - I	14	50	4	4 hr	T
	MATH0803	Operations Research	16	50	4	4 hr	T
	MATH0891	Measure and Probability	18	50	4	4 hr	S
	MATH0892	Mathematical Methods - I and Graph Theory	20	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
III	MATH0901	Partial Differential Equations	22	50	4	4 hr	T
	MATH0902	Functional Analysis	23	50	4	4 hr	T
	MATH0903	Elective - I (E - I) *	3	50	4	4 hr	T
	MATH0991	Mathematical Methods - II and Number Theory	24	50	4	4 hr	S
	MATH0992	Project **	3	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
IV	MATH1001	Algebra - III	26	50	4	4 hr	T
	MATH1002	Dynamical Systems	27	50	4	4 hr	T
	MATH1003	Elective - II (E - II) *	3	50	4	4 hr	T
	MATH1091	Mathematical Computing with Python	29	50	4	4 hr	S
	MATH1092	Dissertation **	3	50	4	4 hr	S
		Total		**250**	**20**	**20 hr**	
		Grand Total		**1000**	**80**		

† In Course Type, ‘T’ stands for Theory and ‘S’ stands for Sessional papers. The methods of assessments for Theory and Sessional papers are as follows:

- **Theory: Internal Assessment (15 marks) + Semester Examination (35 marks)**
- **Sessional: Continuous evaluation throughout the semester.**

Options available for Elective - I and Elective - II Courses*

Elective	Course ID	Name of the Courses	Page Number	Full Marks	Credit Point	Classes per week
I	MATH0903A1	Topology-II	31	50	4	4 hr
	MATH0903A2	Advanced Complex Analysis	33	50	4	4 hr
	MATH0903B1	Special Theory of Relativity	35	50	4	4 hr
	MATH0903B2	Qualitative Theory of Planar Vector Fields - I	36	50	4	4 hr
	MATH0903B3	Advanced Operations Research - I	38	50	4	4 hr
	MATH0903B4	Mathematical Biology - I	40	50	4	4 hr
	MATH0903B5	Advanced Numerical Analysis - I	41	50	4	4 hr
II	MATH1003A1	Operator Algebra	42	50	4	4 hr
	MATH1003A2	Geometry - II	43	50	4	4 hr
	MATH1003A3	Abstract Harmonic Analysis	45	50	4	4 hr
	MATH1003B1	General Theory of Relativity and Cosmology	47	50	4	4 hr
	MATH1003B2	Qualitative Theory of Planar Vector Fields - II	49	50	4	4 hr
	MATH1003B3	Advanced Operations Research - II	50	50	4	4 hr
	MATH1003B4	Mathematical Biology - II	52	50	4	4 hr
	MATH1003B5	Advanced Numerical Analysis - II	53	50	4	4 hr

***N.B. : In E - I and II, exactly *one* from 'MATH0903AX & MATH1003AY' and exactly *one* from 'MATH0903BX & MATH1003BY' will be offered.**

Course Structure

Options available for Project & Dissertation**

Topics for project and dissertation include, but are not limited to, the following:

Lie groups, Lie algebras, Representation Theory, Compact Quantum Groups and Quantum Symmetry, Qualitative Theory of Differential Equations, Dynamical Systems, Complex Dynamics, Ergodic Theory, Riemann Surfaces, Algebraic Graph Theory, Domination in Graphs, Mathematical Cryptography, Cyber Security and Mathematics, Data Science and Analysis with Python, Special Theory of Relativity, General Theory of Relativity, Astrophysics and Cosmology, Theoretical and Observational Cosmology, Mechanics.

Algebra - I

Semester : I	Course Type : T
Course ID : MATH0701	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand group actions and the importance of permutation groups;

CO 2 apply Sylow's theorems in determining groups of a given order; understand the simplicity of alternating groups;

CO 3 understand some advanced concepts in group theory such as semi-direct products, free groups, free products, amalgamated free products, HNN extensions, wreath products, composition series, nilpotent, solvable groups;

CO 4 explain some advanced concepts of ring theory such as principal ideal domain, Euclidean domain, unique factorization domain etc; apply Gauss' theorem, Eisenstein's criterion to check irreducibility of polynomials with coefficients in a unique factorization domain.

Detailed Syllabus

- Group Theory: Review of normal subgroups, quotient groups, and isomorphism theorems; Group actions with examples, orbits and stabilisers, class equations and applications; Lagrange's, Cayley's, Cauchy's and Sylow's theorems in the language of group actions; Symmetric and alternating groups, simplicity of A_n; Direct products and free Abelian groups; Semi-direct products; Composition series, exact sequences; Solvable and nilpotent groups. Free groups; Free products, amalgamated free products, HNN extensions, wreath products.
- Ring Theory: Review of integral domains, ideals, quotient rings and isomorphism theorems, prime and maximal ideals, product of rings, prime and maximal ideals in quotient rings and in finite products, Chinese remainder theorem, field of fractions, irreducible and prime elements, UFD, PID, ED; Polynomial rings, division algorithm, irreducibility criteria, Gauss' theorem; Noetherian rings, Hilbert's basis theorem.

References

[1] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley.

[2] S. Lang, *Algbera*, Springer.

[3] T. W. Hungerford, *Algebra*, Springer.

[4] N. S. Gopalakrishnan, *University Algebra*, Wiley.

[5] Michael Artin, *Algebra*, Prentice Hall.

[6] J. J. Rotman, *An Introduction to the Theory of Groups*, Springer.

[7] D. S. Malik, John M. Mordeson and M. K. Sen, *Fundamentals of Abstract Algebra* McGraw-Hill.

[8] Mahima Ranjan Adhikari and Avishek Adhikari, *Basic Modern Algebra with Applications*, Springer.

[9] Joseph A Gallian, *Contemporary Abstract Algebra*, Brooks/Cole Cengage Learning.

Topology - I

Semester : I	Course Type : T
Course ID : MATH0702	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 explain fundamental concepts of topological spaces such as open sets, closed sets, neighbourhoods, limit points, dense subsets, base, subbase, continuous maps, homeomorphisms, countability axioms, separation axioms, compactness, connectedness, path connectedness, components, path components, locally compact spaces, locally connected and locally path connected spaces;

CO 2 interpret product topology and quotient topology with examples;

CO 3 understand the Urysohn's lemma, Tietz's extension theorem, Urysohn's metrization theorem, Tychnoff theorem, one-point compactification and their applications;

CO 4 describe the basic properties of fundamental groups, covering spaces and their illustrations in determining the fundamental group of the circle.

Detailed Syllabus

- Topological Spaces, Subspace Topology, open and closed sets, neighbourhoods, limit points, interior and closure of a set, dense sets, base and subbase.
- Countability axioms, continuous maps and homeomorphisms.
- Compactness and connectedness, components, path connectedness, locally compact spaces, locally connected spaces, product topology.
- Seperation axioms, regular, completely regular and normal spaces, Urysohn's lemma, Tietz's extension theorem, Urysohn's metrization theorem (statement only), Tychonoff theorem, one-point compactification.
- Topology of pointwise convergence, topology of compact convergence, compact-open topology.
- Quotient spaces with examples (like torus, G/H, Klein's bottle, projective spaces, wedge sum of topological spaces etc.), homotopy, deformation retract, strong deformation retract, contractible spaces.
- Homotopic paths and fundamental group π_1, simply connected topological spaces.
- Covering spaces with examples, path lifting property, homotopy lifting property, computation of $\pi_1(S^1)$, lifting criterion (statement only), deck transformations and properly discontinuous group actions, construction of Universal cover, Galois correspondence for covering spaces.

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill Education.

[2] M. A. Armstrong, *Basic Topology*, Springer.

[3] J. Dugundji, *Topology*, McGraw-Hill Inc., US.

[4] J. Munkres, *Topology, A first course*, Pearson.

[5] J. L. Kelley, *General Topology*, Springer.

[6] J. Munkres, *Elements of Algebraic Topology*, CRC Press.

[7] A. Hatcher, *Algebraic Topology*, Cambridge University Press.

[8] G. E. Bredon, *Topology and Geometry*, Springer.

[9] J. J. Rotman, *Introduction to Algebraic Topology*, Springer.

Ordinary Differential Equations

Semester : I	Course Type : T
Course ID : MATH0703	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 determine the existence and uniqueness of various differential equations; solve homogeneous and inhomogeneous linear differential equations using various methods;

CO 2 solve various boundary value problems constructing Green's functions; apply the Sturm-Liouvelli problem to determine the eigenvalues and eigenfunctions of various boundary value problems that appear in various branches of science;

CO 3 apply the power series method or the Frobenious method to solve various differential equations; construct and apply various special functions, e.g. Legendre, Bessel, hypergeometric, Hermite, etc;

CO 4 investigate the local stability of the critical points of various autonomous systems that appear in the real world.

Detailed Syllabus

- Initial value problems, The Fundamental Existence and Uniqueness Theorem, Maximal interval of existence.
- Linear second order ordinary differential equation with variable coefficients: Recapitulation of the basic theory; Separation theorem and Comparison theorem with applications. Exact equations and self-adjoint operator. Boundary value problems and Lagrange identity. Boundary value problems and Green's functions; Construction of Green's functions, properties and applications. Sturm-Liouville Problems; Eigenfunctions expansion, orthogonality of eigenfunctions, completeness of the eigenfunctions.
- Special Functions: Recapitulation of singular points, points at infinity, series solution and Frobenius method. Hypergeometric equation and functions; Confluent hypergeometric functions and properties with applications. Hermite polynomials. Bessel's functions of first and second kinds, normal form of the Bessel's equation, orthogonality of Bessel functions, Bessel-Fourier series. Legendre equation, Legendre functions, orthogonality of Legendre functions and Legendre series.
- Basic introduction to autonomous systems, phase portraits, isoclines, critical points, stability of the critical points, linearization about a critical point.
- Visualization of some solutions using any mathematical application software like Matlab/Maple/ Mathematica/Python.

References

[1] Lawrence Perko, *Differential Equations and Dynamical Systems*, Springer.

[2] G. F. Simmons, *Differential Equations with applications and historical notes*, CRC Press.

[3] A. C. King, J. Billingham and S. R. Otto, *Differential Equations*, Cambridge University Press.

[4] G. Birkhoff, G-C Rota, *Ordinary Differential Equations*, Wiley and Sons.

[5] Carmen Chicone, *Introduction to ordinary differential equations*, Springer-New York.

[6] R. P. Agarwal and D. O'Regan, *Introduction to ordinary differential equations*, Springer.

[7] E. A. Coddington and N. Levinson, *Theory of ordinary differential equation*, McGraw Hill.

[8] A. Chakraborty, *Elements of ordinary differential equations and special functions*, New Age India International.

Classical Mechanics

Semester : I	Course Type : S
Course ID : MATH0791	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 implement the Newtonian mechanics in various problems related to mathematics, physics and engineering;

CO 2 understand the concepts of constrained motion; derive the Lagrange's equations of motion with the use of D'Alemdert's principle and the principle of least action; use the concept of Hamiltonian in mechanics; apply the canonical transformations and Liouville's theorem in various mechanical problems;

CO 3 understand the central force problem and its application to the planetary motion; signify conserved quantities using Noether's theorem in various mechanical problems;

CO 4 solve several dynamical problems using known mathematical packages.

Detailed Syllabus

- Review of Newtonian mechanics for a single particle and a system of particles; simple illustrations of Newton's equation of motion.
- Constraints and their classification, degrees of freedom, generalized coordinates, D' Alembert's principle, Lagrange's equation of motion for a system of holonomic constraints using D' Alembert's principle (differentiable principle) and Hamilton's principle (integral principle); Applications of the Lagrangian formulation; Conservation theorems; Central force problem.
- Hamilton's equations of motion; cyclic coordinates and their consequences, Routhian, Canonical transformations, Examples of canonical transformations; Poisson's brackets; Liouville's theorem; Hamilton Jacobi theory; Action angle variables; Small oscillations; Noether's theorem.
- Canonical perturbation theory; Preliminaries of rigid body dynamics, Euler's angles.
- Visualization of some dynamical problems using any mathematical application software like Matlab/Maple/
Mathematica/Python.

References

[1] H. Goldstein, *Classical Mechanics*, Addison-Wesley Publishing Company.

[2] N. C. Rana and P. S. Joag, *Classical Mechanics*, Tata McGraw-Hill Education.

[3] L. D. Landau and E. M. Lifshitz, *Mechanics*, Butterworth Heinemann.

[4] S. T. Thornton and J. B. Marion, *Classical Dynamics of Particles and Systems*, Belmont, CA : Brooks/Cole.

[5] E. T. Whittaker, *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies with an introduction to the problem of three bodies*, Cambridge University Press.

[6] R. P. Feynmann, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics: Vol 1, Vol 2, Vol 3* [1], Addison-Wesley Publishing Company.

[1] These books are optional for reading, however, we kept them in the list because we are quite sure that if you start reading them you will a lot.

Complex Analysis

Semester : I	Course Type : S
Course ID : MATH0792	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 conceptualize the notion of complex differentiability and holomorphic functions;

CO 2 understand and compute complex integrals;

CO 3 identify different kinds of singularities and apply various techniques of meromorphic functions;

CO 4 illustrate the role of Morera's theorem and to prove Riemann mapping theorem.

Detailed Syllabus

- Holomorphic functions and the Cauchy-Riemann equations.
- Power series, Analytic Functions, Exponential, Logarithmic and Trigonometric functions, Branch of a complex logarithm.
- Complex integration, Goursat's theorem, Cauchy's integral formula, power series representation, zeros of an analytic function, Liouville's theorem, index of a closed curve, homotopy version of Cauchy's theorem, invariance of integrals under homotopy, Different versions of Cauchy's theorem using homotopy.
- Identity theorem of holomorphic functions, Morera's theorem, sequence of holomorphic functions.
- Classification of singularities, meromorphic functions and residue calculus, Laurent series, contour integration.
- Argument principle, Rouché's theorem, open mapping theorem, maximum modulus principle.
- Möbius transformation, classification of Möbius transformations (elliptic, hyperbolic, parabolic), conformal mapping, Schwarz lemma, conformal automorphisms of disc, upper half plane, complex plane, Riemann sphere.
- Space of continuous functions, normal families, Arzela-Ascorli theorem, compactness and convergence in the space of analytic functions, Montel's theorem, space of meromorphic functions, Riemann mapping theorem.
- (Optional) Infinite product and Weierstrass factorization theorem.
- (Optional) Little Picard theorem and Great Picard theorem.

References

[1] J. B. Conway, *Functions of One Complex Variable*, Narosa Publishing House.

[2] E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press.

[3] L. V. Ahlfors, *Complex Analysis*, McGraw-Hill Education.

[4] T. W. Gamelin, *Complex Analysis*, Springer.

[5] W. Rudin, *Real and Complex Analysis*, McGraw-Hill Education.

[6] S. G. Krantz, *Complex Analysis: The Geometric Viewpoint*, The Mathematical Association of America.

Algebra - II

Semester : II	Course Type : T
Course ID : MATH0801	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the semi-direct products in group theory and learn nilpotent, solvable groups;

CO 2 understand the geometry of inner product spaces and applications of spectral theory; interpret the bilinear forms, alternating forms, and their classifications;

CO 3 explain the fundamental concepts of modules over a commutative ring, free modules; apply classification theorem of finitely generated modules over a PID on finite abelian groups and linear transformations; understand fundamental concepts of algebraic number fields, infinite fields of positive characteristic;

CO 4 apply finite fields to solve various numerical problems.

Detailed Syllabus

- Quick review of Linear Algebra: Vector spaces, linear transformation, matrix of a linear transform, Dual space and double dual.
- Inner-product spaces, Gram-Schmidt orthogonalisation, bi-linear forms, definition of unitary, hermitian, normal, real symmetric and orthogonal linear transformations, spectral theorems; multi-linear forms, alternating forms
- Modules over commutative rings, examples: vector spaces, commutative rings, $\mathbb{Z}$ modules, $F[X]$-modules; submodules. Quotient modules, homomorphisms, isomorphism theorems, $Hom_R(M, N)$ for R-modules M, N, generators and relations for modules, direct products and direct sums, direct summands, free modules, finitely generated modules.
- Field Theory: Field extensions, finite and algebraic extensions, algebraic closure, splitting fields, normal extensions, separable, inseparable and purely inseparable extensions, finite fields, ruler and compass constructions.

References

[1] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley.

[2] S. Lang, *Algbera*, Springer.

[3] T. W. Hungerford, *Algbera*, Springer.

[4] K. Hoffman and R. Kunze, *Linear Algebra*, Prentice-Hall, Inc.

[5] Mahima Ranjan Adhikari and Avishek Adhikari, *Basic Modern Algebra with Applications*, Springer.

Geometry - I (Differential Geometry)

Semester : II	Course Type : T
Course ID : MATH0802	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 interpret fundamental concepts of differentiable manifolds; illustrate vector fields, integral curves, differential forms and exterior derivatives;

CO 2 understand basic concepts of Riemannian manifolds such as Riemannian metrics, isometries, Levi-Civita connection, geodesics, exponential maps, complete Riemannian manifolds; elucidate Hopf-Rinow theorem;

CO 3 understand Riemannian curvature tensor field, sectional curvature of Riemannian manifolds;

CO 4 compute explicitly isometry groups, geodesics, and sectional curvatures of some model spaces like Poincaré upper half plane and its disc model, hyperbolic n-space, Euclidean n-space, n-sphere.

Detailed Syllabus

- Manifolds, smooth structure, smooth manifolds with examples ($\mathbb{R}^n, \mathbb{C}^n$, $\mathbb{S}^n$, $\mathbb{R}P^n$, $\mathrm{GL}(n, \mathbb{R})$, product manifolds etc.), smooth mappings and diffeomorphisms with examples.
- Tangent and cotangent spaces, Jacobian matrix, tangent and cotangent bundles; vector fields, integral curves and Lie brackets, flow of a vector field.
- Submanifolds; regular and critical points of a smooth map, immersion, submersion and embeddings. Differential forms and exterior derivatives.
- Riemannian metric and Riemannian manifolds, length of a smooth curve in a Riemannian manifold, Isometries. Affine connections and covariant derivative, parallel transport, Riemannian connection.
- Geodesics and geodesic flow, the exponential map, normal neighbourhood, connected Riemannian manifolds as metric spaces, geodesics minimizing distance locally, Hopf-Rinow theorem.
- Some model spaces like n-sphere S^n, Poincaré upper half plane $\mathbb{H}^2$, disc model of the Poincaré upper half plane, the hyperbolic n-space $\mathbb{H}^n$.
- Torsion tensor field and Riemannian curvature tensor field, the structural equations and its applications. Sectional curvature of a Riemannian manifold, sectional curvature of $\mathbb{R}^n, \mathbb{S}^n, \mathbb{H}^n$. Riemannian manifolds of constant sectional curvature.

References

[1] F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer.

[2] N. J. Hicks, *Notes on Differential Geometry*, Van Nostrand.

[3] J. L. Dupont, *Differential Geometry*, Aarhus Universitet Matematisk Institut, (https://data.math.au.dk/publications/ln/1993/imf-ln-1993-62.pdf).

[4] M. P. Do Carmo, *Riemannian Geometry*, Birkhäuser.

[5] Gallot, Hulin, Lafontaine, Riemannian Geometry, Universitext-Springer.

[6] J. M. Lee, *Riemannian Manifolds An Introduction to Curvature*, Springer.

[7] L. Tu, *Differential Geometry*, Springer

[8] S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces*, AMS.

Operations Research

Semester : II	Course Type : T
Course ID : MATH0803	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 gain insights in managerial decision to choose the best possible course of action to optimize resource allocation of a real-life problem keeping in mind the linear constraints involved: this has useful application in logistics and economical systems;

CO 2 understand the dual nature of real-life problems and how to utilise the duality to solve a given problem more easily; develop and analyse multi-objective and multi-stage programming, goal programming and dynamic programming, quadratic programming;

CO 3 formulate and solve some real life problems using integer programming, unconstrained and constrained nonlinear programming and queuing problems;

CO 4 apply learned techniques in agricultural planning, biotechnology, data analysis, distribution of goods and resources, emergency and rescue operations, engineering etc.

Detailed Syllabus

- Introduction to OR: Origin of OR and its definition. Concept of optimizing performance measure, Types of OR problems, Deterministic vs. Stochastic optimization, Phases of OR problem approach – problem formulation, building mathematical model, deriving solutions, validating model, controlling and implementing solution.
- Linear programming: Examples from industrial cases, formulation & definitions, Simplex methods, bounded-variables algorithm, Duality, formulation of the dual problem, primal-dual relationships, Revised simplex algorithm, Sensitivity analysis.
- Transportation problem: mathematical formulation, north-west-corner method, least cost method and Vogel's approximation method for determination of starting basic solution, algorithm for solving transportation problem. Assignment problem: mathematical formulation, Hungarian method for solving assignment problem, Travelling Salesman Problem.
- Integer Programming: Standard form, the concept of cutting plane, Gomory's all integer cutting plane method, Gomory's mixed integer method, Branch and Bound method.
- Nonlinear Programming: Introduction to nonlinear programming, Convex function and its generalization, Unconstrained and constrained optimization, Method of Lagrange multiplier, KKT necessary and sufficient conditions for optimality.
- Queuing Theory: Definitions – queue (waiting line), waiting costs, characteristics (arrival, queue, service discipline) of queuing system, queue types (channel vs. phase), Kendall's notation, Little's law, steady state behaviour, Poisson's Process & queue, Models with examples - M/M/1 and its performance measures; M/M/C and its performance measures; brief about some special models (M/G/1).
- Brief introduction to multi-objective and multi-stage programming, Goal Programming and Dynamic Programming.

References

[1] Frederick S. Hillier, Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill Education.

[2] H. S. Taha, *Operations Research*, Pearson Education.

[3] A. Ravindran, Don T. Phillips, James J. Solberg, *Operations Research: Principles and Practice*, Wiley.

[4] Sasieni Maurice, Arther Yaspan, Lawrence Friedman, *OR methods and Problems*, Wiley.

[5] S. D. Sharma, *Operations Research*, Kedar Nath.

[6] John F Shortle, James M Thompson, Donald Gross, Carl M Harris, *Fundamentals of Queueing Theory*, Fifth Edition, Wiley.

[7] T. L. Saaty, *Elements of Queueing Theory, with Applications*, Dover Publications Inc.

[8] B. R. K. Kashyap and M. L. Chaudhry, *Introduction to queueing theory*, Aarkay Publications.

Measure and Probability

Semester : II	Course Type : S
Course ID : MATH0891	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 demonstrate knowledge of Lebesgue measure, Lebesgue measurable sets, Lebesgue measurable functions and Lebesgue integration;

CO 2 describe an understanding of σ-algebra, abstract Measure space, different extension theorems and completion of a measure space; illustrate measures and integrals on product spaces, signed measures, the Radon-Nikodym theorem and its applications; provide an understanding on the properties of L_p-spaces and the fundamental theorem for Lebesgue integral;

CO 3 understand random variables, various types of distributions associated with random variables, moments and modes of convergence in probability theory and apply them in problems;

CO 4 describe an understanding of the Laws of Large Numbers, Central Limit Theorem and their applications.

Detailed Syllabus

Measure Theory

- Algebra, σ-algebra, Monotone Class Theorem, Measure Spaces.
- Outer Measures, Caratheodory Extension Theorem, Pre-measures, Hahn-Kolmogorov Extension Theorem, Uniqueness of the Extension, Completion of a Measure Space.
- Lebesgue Measure and Its Properties.
- Measurable Functions and Their Properties, Modes of Convergence.
- Integration, Monotone Convergence Theorem, Fatou's Lemma, Dominated Convergence Theorem.
- Product Measures, Fubini's Theorem.
- L_p-spaces, Reisz Representation Theorem.
- Signed Measure, The Radon-Nikodym Theorem and Its Applications.
- Fundamental Theorem of Calculus for Lebesgue Integrals.

Probability Theory

- Probability Measure, Probability Space, Continuity Properties of Probability Measure, Random Variables, Probability Distribution of a Random Variable, Functions of a Random Variable and their Probability Distributions.
- Moments, Moment Inequalities (Markov, Chebycheff, Lyapunov and Jensen's inequalities), Moment Generating Function.
- Random Vectors, Probability Distribution of a Random Vector, Functions of Random Vectors and Their Probability Distributions, Independence.
- Characteristic Function and Its Properties, Uniqueness Theorem, Inversion Theorem, Lévy's Continuity Theorem and Bochner's Theorem (Without Proof).
- Sequence of Random Variables, Convergence in Distribution, Convergence in Probability, Almost Sure Convergence, Convergence in rth Mean, Weak and Strong Law of Large Numbers, Borel-Cantelli lemma, Limiting Characteristic Function, Classical Central Limit Theorem, Lindeberg & Lyapunov Central Limit Theorems (Without Proof), Applications of the Central Limit Theorems.

References

[1] T. Tao, *An Introduction to Measure Theory*, American Mathematical Society.

[2] I. K. Rana, *An Introduction to Measure and Integration*, Narosa.

[3] P. R. Halmos, *Measure Theory*, Springer.

[4] H. L. Royden, *Real Analysis*, Pearson.

[5] W. Rudin, *Real and Complex Analysis*, McGraw Hill Education.

[6] P. Billingsley, *Probability and Measure*, Wiley.

[7] A. Gut, *Probability: A Graduate Course*, Springer.

[8] R. G. Laha and V. K. Rohatgi, *Probability Theory*, Dover Publications Inc.

[9] W. Feller, *Introduction to Probability Theory and Its Applications: Vol. 1 and 2*, Wiley.

Mathematical Methods - I and Graph Theory

Semester : II	Course Type : S
Course ID : MATH0892	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 gather knowledge about mathematical concepts of the methods of applied mathematics;

CO 2 solve different types of problems (ODEs & PDEs) by using different methods;

CO 3 understand the application of graph in real-life problems;

CO 4 understand the application of graph in its application in different theorems.

Detailed Syllabus

Mathematical Methods I

- Tensors: Introduction to tensors, tensor algebra.
- Integral Transforms:

  Fourier Transform: Fourier integral theorem, Riemann-Lebesgue lemma, Cosine and sine transforms, inversion theorem, properties of FT with applications, Derivatives, Convolution theorem, convolution of Fourier sine/cosine transform. Application of FT of ODE and PDE.

  Laplace Transform: Functions of exponential order and existence condition for LT. Properties of LT with applications, Inversion of LT, application in solving ODE and PDE. Complex inversion and Bromwich contour integral.

  Mellin Transform: Introduction to Mellin transforms, properties and applications.

  Hankel Transform (if time permits): Introduction, properties and applications.

Graph Theory

- Graphs, Products of Graphs; Connectedness, Trees, Spanning Tree; Degree Sequences: Havel-Hakimi Theorem and its Applications; Connectivity; Eulerian and Hamiltonian graphs: Ore's Theorem, Dirac's Theorem; Clique Number, Chromatic Number: Their Relations: Brooke's Theorem and Perfect Graphs, Domination number, Independence number: Relations and Bounds. Isomorphism of Graphs, Cayley Graphs, Strongly Regular Graphs: Adjacency Matrix of a Graph: Properties and Eigenvalues;
- Visualization of few graph theoretic results using the software SAGEMATH.

References

[1] B. Spain, *Tensor Calculus, a concise course*, Dover Publications, Inc.

[2] L. Brand, *Vector and Tensor Analysis*, John Wiley & Sons.

[3] H. Lass, *Vector and Tensor Analysis*, McGraw-Hill Book Company, Inc.

[4] I. N. Sneddon, *Use of Integral Transforms*, McGraw Hill.

[5] H. G. ter Morsche, J. C. van den Berg, E. M. van de Vrie, *Fourier and Laplace Transforms*, Cambridge University Press.

[6] I. N. Sneddon, *Fourier Transform*, Dover Publications.

[7] R. N. Bracewell, *Fourier Transform and its Applications*, McGraw Hill.

[8] J. L. Schiff, *Laplace Transform Theory and Applications*, Springer.

[9] D. B. West, *Introduction to Graph Theory*, Pearson.

[10] C. Godsil and G. Royle, *Algebraic Graph Theory*, Springer-Verlag.

[11] R. Diestel, *Graph Theory*, Springer.

Partial Differential Equations

Semester : III	Course Type : T
Course ID : MATH0901	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand and solve linear and nonlinear first order PDE by various methods;

CO 2 solve second order linear PDEs including heat equations, wave equations and Laplace equations;

CO 3 apply Duhamel's principle to solve PDE;

CO 4 apply the Representation theorem and Green's function in PDE;

Detailed Syllabus

- Recapitulation of the basic definition of a general PDE of order m on a n dimensional space, classifications. Formation of PDE, general solution, complete integral and singular solutions. Lagrange's and Charpit's method with geometrical interpretation. General first order linear and nonlinear PDEs, method of characteristics, Cauchy problem, non characteristic condition.
- Second order PDEs, canonical forms and classifications by characteristic, invariance of discriminant.
- Second-order Hyperbolic Equations: One dimensional wave equation and D'Alembert's solution. Spherical Means, Euler-Poisson-Darboux equation, Poisson solution, Kirchoff's solution, Duhamel's principle. Uniqueness of solution: energy methods. Domain of dependence, Range of influence and Causality.
- Second-order Elliptic Equations: Solution by the method of separation of variables and the derivation of the Poisson solution on a disc. Fundamental solution. Mean value Formula, Strong Maximum Principle, Regularity and smoothness of harmonic functions. Liouville's theorem. Green's function and Dirichlet's problem. Green's function derivation with applications in half plane and a disc. Uniqueness of solution.
- Second-order Parabolic Equations: Method of separation of variables. Fundamental solution and heat kernel. Poisson formula. Solution of the inhomogeneous heat equation. Uniqueness of solution: energy methods.
- Visualization of some solutions using any mathematical application software like Matlab/Maple/Mathematica/Python.

References

[1] L. C. Evans, *Partial Differential Equations*, American Mathematical Society.

[2] I. N. Sneddon, *Elements of Partial Differential Equations*, Dover Publications.

[3] P. Prasad, R. Ravindran, *Partial Differential Equations*, New Age India International Publishers.

[4] V. I. Arnold, *Lectures on Partial Differential Equations*, Springer.

[5] J. Fritz, *Partial Differential Equations*, Springer.

Functional Analysis

Semester : III	Course Type : T
Course ID : MATH0902	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand Normed linear spaces, Banach Spaces with examples and Properties, Riesz Lemma and its application, Equivalence of norm and its consequences; bounded Linear operator on infinite dimensional normed linear space;

CO 2 apply Hahn-Banach Theorem, Open-Mapping and Closed graph Theorem, Uniform-Boundedness Principle;

CO 3 understand Weak and Weak* topology and Banach-Alaoglu Theorem; Hilbert Space, Orthonormal sets, Bessel's inequality, Riesz representation Theorem;

CO 4 apply bounded linear operators on Hilbert Space, concentrate on special types of operators for instance projection, self-adjoint, unitary and normal operators.

Detailed Syllabus

- Normed linear spaces, Banach spaces, Examples and elementary properties, Equivalence of norm, Riesz Lemma and its applications, Review of Baire Category Theorem and its consequences regarding the dimension, Bounded linear operators.
- Hahn-Banach Theorem and its consequences, Hahn-Banach separation Theorems, Banach-Steinhaus theorem, Open mapping theorem, closed graph theorem and its applications.
- Dual space, Computing duals of l^p, L^p and $C[0,1]$, reflexive space and its properties.
- Weak and weak* topology, Schur lemma, Banach-Alaoglu Theorem.
- Hilbert spaces, orthonormal sets, projection theorem, Bessel's inequality, Parseval's identity, Riesz representation theorem.
- Bounded operators on a Hilbert space, adjoint of an operator, self-adjoint operator, unitary and normal operators, projection, spectrum and spectral radius of a bounded operator, Compact operator.
- Review of spectral theorem in finite dimensional Hilbert space, Spectral theorem for compact, self-adjoint operators.

References

[1] J. B. Conway, *A course in Functional Analysis*, Springer.

[2] Walter Rudin, *Functional Analysis*, McGraw Hill.

[3] Kosaku Yosida, *Functional Analysis*, Springer.

[4] B. V. Limaye, *Functional Analysis*, New Age International Publisher.

[5] R. Bhatia, *Notes on Functional Analysis*, Hindustan Book Agency.

Mathematical Methods - II and Number Theory

Semester : III	Course Type : S
Course ID : MATH0991	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply the calculus of variations in problems related to Hamilton's Principle, Euler-Lagrange's equations;

CO 2 understand the techniques for solving integral equations involving the Fredholm and Volterra Integral Equations and Resolvent Kernels, Symmetric Kernels;

CO 3 know about the unit of groups, primitive roots; compute Quadratic Residues and Legendre symbols along with their uses.

CO 4 apply the above knowledge in primality testing and cryptology.

Detailed Syllabus

Mathematical Methods II

- Calculus of Variations: The brachistochrone problem, Hamilton's principle, some variational problems from geometry, extrema of functionals, Euler-Lagrange equations, some special cases of the Euler-Lagrange equations.
- Integral Equations: Definition and classifications. Solution by separable kernels. Approximate method and Neumann series. Fredholm alternative theorem. Resolvent kernel and applications. Conversion of IVP and BVP to integral equations, Green's function. Symmetric kernels and bilinear forms. Hilbert-Schimdt theorem and applications. Symmetric integral equation.
- Visualization of some solutions using any mathematical application software like Matlab/Maple/ Mathematica/Python.

Number Theory

- The Arithmetic of $\mathbb{Z}_p$, p a prime, pseudo prime and Carmichael Numbers, Fermat Numbers, Perfect Numbers, Mersenne Numbers.
- Primitive roots, the group of units of $\mathbb{Z}_n$, the existence of primitive roots.
- Quadratic residues and non quadratic residues, Legendre symbol, proof of the law of quadratic reciprocity, Jacobi symbols.
- Primality Testing, Miller-Rabin test, Solovay Strassen test.
- Application of number theory in Cryptography, specially in Public Key Cryptography such as RSA and ElGamal Public Key Cryptographic schemes. Few attacks on RSA PKC, DLP and Diffie Hellman Key Exchange Protocol.
- Visualization of few number theoretic results using the software SAGEMATH.

References

[1] Bruce van Brunt, *The Calculus of Variations*, Springer.

[2] U. Brechtken-Manderscheid, *Introduction to the Calculus of Variations*, Springer Science+Business Media, B.V.

[3] M. L. Krasnov, G. I. Makarenko and A. I. Kiselev, *Problems and exercises in the Calculus of Variations*, Mir Publishers.

[4] Robert Weinstock, *Calculus of Variations with applications to Physics and Engineering*, Dover Publications.

[5] R. P. Kanwal, *Linear Integral Equations: Theory and Techniques*, Birkhauser.

[6] F. G. Tricomi, *Integral Equations*, Dover Publications.

[7] S. G. Mikhlin, *Linear Integral Equations*, Dover Publications.

[8] D. M. Burton, *Elementary Number Theory*, Wm. C. Brown Publishers, Dulreque, Lowa, 1989.

[9] Gareth A Jones and J Mary Jones, *Elementary Number Theory*, Springer International Edition.

[10] Richard A Mollin, *Advanced Number Theory with Applications* CRC Press, A Chapman & Hall Book.

[11] Mahima Ranjan Adhikari and Avishek Adhikari, *Basic Modern Algebra with Applications*, Springer.

[12] Kenneth. H. Rosen, *Elementary Number Theory and Its Applications* AT&T Bell Laboratories, Addition Wesley Publishing Company.

Algebra - III

Semester : IV	Course Type : T
Course ID : MATH1001	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand some advanced concepts in module theory such as tensor product and exact sequences in modules;

CO 2 understand the classification theorem of finitely generated modules over a PID and its applications on finite abelian groups and linear transformations;

CO 3 understand Galois extensions and explicit computations of Galois groups;

CO 4 understand fundamental theorem of Galois theory and its applications to explicitly determine the correspondence between subgroups of the Galois group and intermediate fields of a Galois extension; the solvability of radicals and its applications to determine the nature of roots of polynomial equations.

Detailed Syllabus

- Tensor product of modules: definition, universal property, 'extension of scalars', basic properties and elementary computations.
- Exact sequences of modules: Projective, injective and flat modules (only definitions and examples).
- Noetherian modules, torsion and annihilator submodules, finitely generated modules over PID, structure theorems for modules over PID: existence (invariant factor form & elementary divisor form) and uniqueness, primary decomposition theorem.
- Applications: (a) to modules over $\mathbb{Z}$: fundamental theorem of finitely generated abelian groups; (b) to modules over $F[X]$: Canonical forms - Rational and Jordan canonical forms.
- Galois theory: Galois extensions and Galois groups, fundamental theorem of Galois theory; Examples, explicit computation and applications of Galois theory; Roots of unity, cyclotomic extensions, construction of regular n-gons, solvability by radicals, quintics are not solvable by radicals.

References

[1] D. S. Dummit and R. M. Foote, *Abstract Algebra*, Wiley.

[2] S. Lang, *Algbera*, Springer, GTM.

[3] David A. Cox, *Galois Theory*, Wiley.

[4] Ian Stewart, *Galois Theory*, Chapman & Hall/CRC.

[5] Joseph Rotman, *Galois Theory*, Springer.

Dynamical Systems

Semester : IV	Course Type : T
Course ID : MATH1002	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of the course, a student will be able to:

CO 1 demonstrate the knowledge on classification of linear flows on $\mathbb{R}^n$ up to C^0 conjugacy and C^0 equivalency; investigate the behavior of flows in a neighborhood of a singularity with the help of local stable and center manifold theorems;

CO 2 illustrate the limit sets with a specific reference to the nature of limit sets of flows in a compact subset of $\mathbb{R}^2$; characterize the nature of flows in a neighborhood of invariant sets;

CO 3 predict the long term behaviour of some families of self maps on (locally) one dimensional spaces like real line, circle etc;

CO 4 learn the tools of topological conjugacy, mixing, transitivity, topological entropy, chaos, bifurcation theory, Schwarzian derivative and Hausdorff dimensions to have further insight on the dynamics of a self map.

Detailed Syllabus

Continuous Dynamical Systems

- Vector Fields and Flows on $\mathbb{R}^n$, Topological (C^0) Conjugacy and Equivalency, Classification of Linear Flows up to C^0 Conjugacy and Equivalency.
- α & ω Limit Sets of an Orbit, Attractors, Periodic Orbits and Limit Cycles.
- Local Structure of Critical Points (The Local Stable Manifold Theorem, The Hartman-Grobman Theorem, The Center Manifold Theorem), Lyapunov Function.
- Periodic Orbits, The Poincaré Map and Floquet Theory, The Poincaré-Benedixson Theorem, Dulac's Criteria.
- Chaotic Attractors, Lyapunov Exponents, Test for Chaotic Attractors.

Discrete Dynamical Systems

- Examples of discrete dynamical systems, iterations of functions, phase portraits, periodic points and stable sets, differentiability and its implications, attracting/repelling/neutral periodic points, graphical analysis, cobweb diagram, Newton's method as an iterative process.
- Circle maps, rotation number, periodic points of circle maps, Poincaré classification theorem, devil's staircase, Denjoy's example.
- Sarkovskii's theorem and Sarkovskii ordering.
- Limit sets and recurrence, topological conjugacy, topological transitivity, topological mixing, Devaney chaos, topological entropy, structural stability.

- Quadratic family and logistic family, symbolic dynamics, subshifts and codes, subshifts of finite type (SFT), Perron-Frobenius theorem, topological entropy and the Zeta function of an SFT.
- Schwarzian derivative and bound on the number of attracting periodic orbits.
- Bifurcation theory, classification of bifurcations, period doubling cascade, chaos at the end of bifurcation diagram.
- Hausdorff measure and Hausdorff dimension, space-filling curve, iterated function system and fractals.
- (Optional) Dynamics of linear maps, the horseshoe map, hyperbolic toral automorphisms.

References

[1] C. Robinson, *An Introduction to Dynamical Systems: Continuous and Discrete*, AMS.

[2] L. Perko, *Differential Equations and Dynamical Systems*, Springer.

[3] C. Robinson, *Dynamical Systems: Stability, Symbolic Dynamics and Chaos*, CRC Press.

[4] R. L. Devaney, *An Introduction to Chaotic Dynamical Systems*, CRC Press.

[5] M. Brin and G. Stuck, *Introduction to Dynamical Systems*, Cambridge University Press.

[6] Y. Pesin and V. Climenhaga, *Lectures on Fractal Geometry and Dynamical Systems*, AMS.

[7] A. Katok and B. Hasselblatt, *Introduction to the Modern Theory of Dynamical Systems*, Cambridge University Press.

[8] M. F. Barnsley, *Fractals Everywhere*, Academic Press Professional.

[9] K. Falconer, *Fractal Geometry: Mathematical Foundations & Applications*, Wiley.

Mathematical Computing with Python

Semester : IV	Course Type : S
Course ID : MATH1091	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand how to use lists, tuples, and dictionaries in Python programs;

CO 2 use indexing and slicing to access data, write loops and decision statements and write functions and pass arguments in Python programs;

CO 3 understand the use of library functions in Python; read and write files in Python;

CO 4 implement a few real life applications.

Detailed Syllabus

- **Introduction to Basic Computing:** Basics of Instruction Cycle (fetch-execute cycle). Idea of memory, CPU and GPU.
- **Introduction to Programming Languages:** Types of programming languages. Compiled and Interpreted(Scripting) Languages. Statically typed and Dynamically typed programming languages.
- **Introduction to Python:** Downloading and installing Python. Understanding, how to use Python and PIP(Package Installer for Python). Understanding the usage of Python terminal interpreter. Execution of python script (with basic "hello world" program). Installing and using IPython with Jupyter Notebook. (One can use Kaggle or Google Colab)
- **Basics of Python:**
  Learning the fundamentals of Python programming.
  1. Hello World(Printing)
  2. Indentation, Comments
  3. Built In Data-Types: *int, float, complex str, bool, set, dict*
  4. Iterators: *list, range, str*
  5. Control Flow: Sequential, Decision(if-else, nested if-else), Repetition (for-loop, while-loop).
  6. Function: Function definition, Parameters, Arguments, Local variables, Calling a Function, Built-In Python Functions (abs(), any(), bin(), bytes(), chr(), com(), float(), format(), input(), int(), len(), list(), max(), min(), open(), pow(), print(), str(), sum() etc.).
  7. Python Strings: Replace, Join, Split, Reverse, Uppercase, Lowercase, etc. Use of Len(), index(), find(), join() etc.
  8. File Handling: Opening and manipulating text file, binary files, csv file. Basics of folder manipulation.
  9. Basics of OOPs, objects and methods. Custom data types.
- **Packages and Modules:** What is a Python Library? Learn to use the documentation.

1. Numpy: Fields of usage. Array, ndarray*, dot product of arrays (real and complex), matrix, product of matrix, transpose of matrix, inverting matrix, finding eigenvalues, singular value decomposition*, mathematical functions in numpy.
2. Matplotlib: Drawing basic graphs. Drawing graphs from data (scatter, line, pi-chart, bar-chart). Reading and manipulating images.
3. Pandas (if time permits): I/O of different files (csv, excel file). Basics of DataFrame* and Series Object. Basic Data Analysis and cleaning of Data. Conversion of data types, indexing and iteration of data types.

- **Applications in Basic Mathematics:**
  1. Applications in solving linear and nonlinear ODE (*Runge-Kutta method, shooting methods etc.*).
  2. Applications in evaluating single and multiple integrals (*Trapezoidal, Simpson's, Gaussian Quadrature etc.*).
  3. Application in finding roots of non-linear/transcendental algebraic equations (*Bisection method, Newton's method, fsolve etc.*).
  4. Applications in Number Theory: Finding Quadratic Residues, Jacobi Symbols, Probabilistic Primality testing such as Solovay Strassen Algorithm.
- **Applications to Data Science and Machine Learning (ML):** What is Data Science? Usefulness of Data Science, Objective of Machine Learning. Idea of test, train dataset. Classification of ML. Supervised and Unsupervised Learning (mentions of reinforcement learning, transfer learning). Linear Regression, Logistic Regression, Decision Tree, idea of Support Vector Machine.
- **Applications to Cyber Security:** Implementations of various cryptographic primitives such as Public key cryptosystem, Signature Scheme, Secret Sharing, Hash function, Stream Ciphers etc.
- Artificial Neural Network. Back Propagation. Different types of neural network (RNN, CNN etc.).

References

[1] Vernon L. Ceder, *The Quick Python Book*, Second Edition, Manning, 2010.

[2] J. C. Bautista, *Mathematics and Python Programming*, Lulu.com, 2014.

[3] Amit Saha, *Doing Math with Python, No Starch Press*, San Francisco, 2015.

[4] Alex Martelli, Anna Ravenscroft, Steve Holden, *Python in a Nutshell*, 3rd Edition, O'Reilly Media, Inc, 2017.

[5] Christian Hill, *Learning scientific programming with Python*, Cambridge University Press, 2015.

[6] Alex Gezerlis, *Numerical Methods in Physics with Python*, Cambridge University Press, 2020.

Options for Elective - I

Topology - II

Semester : III	Course Type : T
Course ID : MATH0903A1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply the Seifert-van Kampen theorem to determine explicitly the fundamental groups of various spaces;

CO 2 know simplicial complexes and their basic properties;

CO 3 understand the fundamental concepts of singular homology, homology of long exact-sequences, Mayer-Vietoris sequences and its application in determining the homology groups of n-spheres;

CO 4 know some other homology theory such as cellular homology; understand the Stone-Čech compactification and the rings of continuous functions.

Detailed Syllabus

- Brower fixed-point theorem, Borsuk-Ulam theorem, winding numbers and applications.
- Free abelian groups; Free groups, free products, amalgamated free products and HNN - extensions of groups, Seifert-van Kampen theorem, fundamental groups of closed genus-g and other surfaces, K(G,1) spaces.
- Simplicial complexes, chains and boundary homomorphisms, simplicial homology, examples and computations. Hurewicz theorem: H_1 as the abelianisation of π_1 (explicit illustration through $\pi_1(\Sigma_g)$ and $H_1(\Sigma_g)$).
- Singular homology, chain complexes, homotopoy invariance, equivalence of simplicial and singular homology.
- Relative homology, homology long exact-sequences, excision theorem and applications; computation of degrees of maps between spheres, Mayer-Vietoris sequences and applications.
- CW-complexes, cellular homology, computing homology groups of spaces (like S^n, $\mathbb{R}P^n$, $\mathbb{C}P^n$, lens spaces, closed genus-g surfaces, etc.); Betti numbers and Euler characteristics;
- Nets and filters, Rings of continuous functions, Stone-Čech compactification.
- Homology of groups.

References

[1] J. Kelley, *General Topology*, Springer.

[2] J. Dugundji, *Topology*, UBS Publishers.

[3] L. Gillman and M. Jerison, *Rings of Continuous Functions*, Springer.

[4] J. Munkres, *Topology*, Pearson.

[5] R. C. Walker, *The Stone-Čech compactification*, Springer.

[6] J. Munkres, *Elements of Algebraic Topology*, CRC Press.

[7] A. Hatcher, *Algebraic Topology*, CUP.

[8] M. Greenberg, J. Harper, *Algebraic Topology: A First Course*, The Benjamin/Cummings Publishing Company.

[9] G. Bredon, *Topology and Geometry*, Springer.

[10] W. Fulton, *Algebraic Topology: A First Course*, Springer.

Advanced Complex Analysis

Semester : III	Course Type : T
Course ID : MATH0903A2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 get an idea of a Riemann surface of a function and understand the Hurwitz's theorem, Normality, Montel's theorem, Riemann mapping theorem, Schwarz-Christoffel formula;

CO 2 understand the Marty's theorem, Zalcman's lemma, and Fundamental normality; understand the factorization theorems due to Weierstrass and Hadamard with the order and genus of an entire function; understand the Runge's theorem and its consequences;

CO 3 understand the analytic continuation, Schwarz reflection principle, Monodromy theorem;

CO 4 ideas and application of harmonic, subharmonic and superharmonic functions; understand the proofs of the little and the great Picard's theorem.

Detailed Syllabus

- Conformal Mappings, Level curves, Survey of elementary mappings, Elementary Riemann surfaces.
- Revision of Compactness and convergence in the space of analytic functions, Convergence on compact subsets, Hurwitz's classical version, Normality, Montel's theorem, Riemann mapping theorem, Schwarz-Christoffel formula.
- Weierstrass spherical convergence theorem, spherical metric, spherical derivative, Marty's theorem, Zalcman's lemma, Bloch's principle, Fundamental normality.
- Weierstrass factorization theorem, Factorization of the Sine function, Gamma function, Riemann Zeta function, Jensen's Formula, Genus and order of an entire function, Hadamard factorization theorem.
- Runge's theorem, Simple connectedness, Mittag-Leffler's theorem.
- Analytic continuation and Riemann surfaces, Schwarz reflection principle, Analytic continuation along a path, Monodromy theorem, Sheaf of germs of analytic functions on an open set, Analytic manifolds, Covering spaces.
- Basic properties of harmonic functions, Harmonic functions on a disk, Subharmonic and Superharmonic functions, Dirichlet problem, Green's functions, Harmonic measure.
- Bloch's Theorem, the little and the great Picard's theorem.

References

[1] J. B. Conway, *Functions of One Complex Variable*, Narosa Publishing House.

[2] E. M. Stein and R. Shakarchi, *Complex Analysis*, Princeton University Press.

[3] L. V. Ahlfors, *Complex Analysis*, McGraw-Hill Education.

[4] T. W. Gamelin, *Complex Analysis*, Springer.

[5] W. Rudin, *Real and Complex Analysis*, McGraw-Hill Education.

[6] S. G. Krantz, *Complex Analysis: The Geometric Viewpoint*, The Mathematical Association of America.

Special Theory of Relativity

Semester : III	Course Type : T
Course ID : MATH0903B1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the limitations of the Newtonian mechanics;

CO 2 understand the Lorentz transformations and Lorentz group;

CO 3 understand the concept of length contraction and time dilation; understand the 4-dimensional Minkowskian space-time and its consequences;

CO 4 understand four vectors, relativistic mechanics, equivalence of mass and energy (the famous equation $E = mc^2$).

Detailed Syllabus

- Differentiable manifolds, tensor calculus, partial derivative of a tensor, Lie derivative, affine connection, covariant differentiation, introduction to metric and metric tensor.
- Newton's laws and inertial frames, Galilean transformations, Newtonian relativity, The Michelson-Morley experiment, Einstein's thoughts and his postulates of special theory of relativity.
- The relativity of simultaneity, Lorentz transformations; mathematical properties of Lorentz transformations, spacetime invariant, length contraction, time dilation, twin paradox, relativistic addition of velocities.
- Minkowski's spacetime, space-like, time-like and light-like intervals, lightcone; four vectors, geometry of four vectors, proper time, relativistic mass, momentum and energy, equivalence of mass and energy, energy-momentum tensor.

References

[1] R. Resnick, *Introduction to Special Relativity*, John Wiley & Sons.

[2] A. P. French, *Special Relativity*, CRC Press.

[3] S. Banerjee and A. Banerjee, *The Special Theory of Relativity*, PHI.

[4] Ray D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press.

[5] W. Rindler, *Relativity - Special, general and cosmological*, Oxford University Press

[6] Ta-Pei Cheng, *Relativity, Gravitation and Cosmology*, Oxford University Press

Qualitative Theory of Planar Vector Fields - I

Semester : III	Course Type : T
Course ID : MATH0903B2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of the course, a student will be able to

CO 1 investigate local behaviour of flows associated with vector fields by the local stable and center manifold theorems; characterize limit sets in the plane using the Poincaré-Benedixson theorem; construct the Poincaré map for some flows associated with planar vector fields;

CO 2 derive normal forms for certain singularities of planar vector fields; demonstrate global phase portraits for certain polynomial planar vector fields with the help of the Poincaré compactification;

CO 3 compute the Poincaré index of a singularity and demonstrate the Poincaré-Hopf index theorem;

CO 4 illustrate the essence of integrability of autonomous systems with some algebraic approaches.

Detailed Syllabus

- Basic Results on the Qualitative Theory of Planar Vector Fields: Flows, Singularities of Vector Fields, Phase Portrait, Limit Sets, Stability, The Poincaré Map and The Poincaré-Benedixson Theory.
- Normal Form Theory: Near-Identity Transformations, Normal Forms for Certain Singularities of Vector Fields.
- Desingularization of Non-elementary Singularities: Homogeneous and Quasi-homogeneous Blow up, Desingularization and the Lojasiewicz Property, Nilpotent Singularities.
- Global Phase Portrait: Infinite Singularities, Poincaré and Poincaré-Lyapunov Compactification, Phase Portraits for Global Flows, Separatrix Configurations.
- Index Theory: Index of Singularities of a Vector Field, Hopf's Theorem, Vector Fields on the Sphere $\mathbb{S}^2$, The Poincaré-Hopf Index Theorem.
- Integrability: First Integrals and Invariants, Integrating Factors, Invariant Algebraic Curves, Exponential Factors, Darboux Theory of Integrability, Prelle-Singer and Singer Results, Examples.

References

[1] F. Dumortier, J. Llibre and J. C. Artés, *Qualitative Theory of Planar Differential Systems*, Springer.

[2] L. Perko, *Differential Equations and Dynamical Systems*, Springer.

[3] V. I. Arnold, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer.

[4] X. Zhang, *Integrability of Dynamical Systems: Algebra and Analysis*, Springer.

[5] V. V. Nemytskii and V. V. Stepanov, *Qualitative Theory of Differential Equations*, Princeton University Press.

[6] A. A. Andronov, E. A. Leontovich, I. I. Gordon and A. G. Maier, *Qualitative Theory of Second-Order Dynamic Systems*, Wiley.

[7] A. D. Bruno, *Local Methods in Non-linear Differential Equations*, Springer.

Advanced Operations Research - I

Semester : III	Course Type : T
Course ID : MATH0903B3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 efficiently handle deterministic inventory models and implement it in practical problems;

CO 2 check the robustness of the developed models and make appropriate decisions;

CO 3 efficiently develop and solve sequencing problems;

CO 4 implement game theory in the study of mathematical models of strategic interaction between rational decision-makers and its applications in different fields of social sciences.

Detailed Syllabus

- **Deterministic Inventory control Models:** Nature of inventory problems. Structure of inventory systems. Definition of inventory problem. Important parameters associated with inventory problems. Variables in inventory problems. Controlled and uncontrolled variables. Types of inventory systems and inventory policies. Statistical and dynamical inventory problems. Deterministic inventory models / systems. Harris-Wilson model. Economic lot size systems. Sensitivity of the lot size systems. Order level systems and their sensitivity analysis. Order level lot size and their sensitivity studies. Non-constant demand models under (s, q), (t, si) and (ti, si) policies. Power law and linear travel demand situations. Lot size systems with different cost properties: (i) Quantity discounts, (ii) Price-change anticipation, (iii) Perishable goods system. Multi-item inventory models with (i) single linear restriction, (ii) More than one linear restriction, (iii) non-linear restrictions.
- **Sequencing:** Sequencing problems, Solution of sequencing problems, Processing n jobs through two machines, Processing n jobs through three machines, Optimal solutions, Processing of two jobs through m machines, Graphical method of solution, Processing n jobs through m machines.
- **Game Theory and Decisions Making:** Game theory to determine strategic behavior, Elements of decision theory and decision trees, Elements of cooperative and non-cooperative games, Two-person zero-sum game, Bimatrix games and Lemke's algorithm for solving bimatrix games.

References

[1] John A. Muckstadt, Amar Sapra, *Principles of Inventory Management*, Springer.

[2] Sven Axsater, *Inventory Control*, Springer.

[3] Eliczer Nadder, *Inventory Systems*, John Wiley and Sons.

[4] G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Prentice Hall.

[5] R. J. Tersine and M. Hays, *Principles of Inventory and Material Management*, Pearson.

[6] A. Ravindran, Don T. Phillips, James J. Solberg, *Operations Research: Principles and Practice*, Wiley.

[7] H. S. Taha, *Operations Research*, Pearson Education.

[8] Frederick S. Hillier, Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill Education.

[9] Sasieni Maurice, Arther Yaspan, Lawrence Friedman, *OR methods and Problems*, Wiley.

[10] S. D. Sharma, *Operations Research*, Kedar Nath.

[11] Paul R. Thie, Gerard E. Keough, *An Introduction to Linear Programming and Game Theory*, Wiley-Interscience

Mathematical Biology - I

Semester : III	Course Type : T
Course ID : MATH0903B4	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 formulate mathematical models of various biological and physical systems (interaction of species, epidemiology, environmental modeling etc) in terms of difference equations;

CO 2 develop mathematical models by using ODEs and PDEs;

CO 3 investigate systematically mathematical models of biological systems locally as well as globally;

CO 4 apply the knowledge of mathematics in selected real life situations.

Detailed Syllabus

- Mathematical Biology and the Modeling Process: an Overview.
- Qualitative analysis of continuous models: Steady state solutions, stability and linearization, Routh- Hurwitz Criteria, Phase plane methods and qualitative solutions, Lyapunov second method for stability, bifurcations (saddle-node, transcritical, pitchfork and Hopf).
- Continuous growth functions: Malthus growth, logistic growth, Gompertz growth, Holling type growth. One species models: Different growth models for single species, harvesting of species. Two species models: Equilibria and their stability analysis.
- Epidemic Models: Deterministic model of simple epidemic, Infection through vertical and horizontal transmission, General epidemic- Karmac-Mackendric Threshold Theorem, SI, SIR, SIRS models.
- Discrete system: Overview of difference equations, steady state solution and linear stability analysis, Introduction to Discrete Models, Linear Models, Growth models, Decay models, Discrete Prey-Predator model and Epidemic model.

References

[1] H. I. Freedman, *Deterministic Mathematical Models in Population Ecology*, Marcel Dekker, Inc.

[2] M. Kot, *Elements of Mathematical Ecology*, Cambridge University Press.

[3] D. Alstod, *Basic Populas Models of Ecology*, Prentice Hall, Inc., NJ.

[4] J. D. Murray, *Mathematical Biology - I*, Springer and Verlag.

[5] L. Perko, *Differential Equations and Dynamical Systems*, Springer Verlag.

Advanced Numerical Analysis - I

Semester : III	Course Type : T
Course ID : MATH0903B5	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concepts of various errors in numerical evaluation of mathematical problems; understand various numerical techniques to find the roots of various nonlinear equations;

CO 2 understand and apply various methods of interpolating data points;

CO 3 solve any algebraic system of equations, using direct or iterative techniques;

CO 4 gain the program writing skills in a computer based environment.

Detailed Syllabus

- Errors: Sources and propagation.
- Non linear Equations: Recapitulation of Newton's method, convergence and rate of convergence, Aitken's method and convergence criterion. Applications.
- System of linear equations: Triangular factorization, Iterative methods: Gauss Seidel method and its convergence, Successive Over Relaxation method. Applications
- Approximation of functions: Least square methods, polynomial economization.
- Eigenvalue and Eigenfunctions of a matrix: Power methods, Given's method, Householder method and QR factorization.
- Computer programming and code development using C/Python for Given's method, SOR method, Aitken's method, least square method and visualization of graphical results wherever possible.

References

[1] K. Atkinson, *Introduction to Numerical Analysis*, J. Wiley and Sons.

[2] E. Isaacson and H. B. Keller, *Analysis of Numerical Methods*, Dover Publications.

[3] F. B. Hildebrand, *Introduction to Numerical Analysis*, Dover Publications.

[4] J. Stoer, R. Bulirsch, *Introduction to Numerical Analysis*, Springer Science.

[5] W. Cheney, D. Kincaid, *Numerical Mathematics and Computing*, Brooks/Cole.

[6] William H. Press, Brian P. Flannery, Saul Teukolsky, William T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press.

Options for Elective - II

Operator Algebra

Semester : IV	Course Type : T
Course ID : MATH1003A1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand Banach Algebra with Examples and properties, Spectrum and its properties, Spectral radius formula;

CO 2 understand the character space and connection with maximal ideals, Gelfand Transformation;

CO 3 understand C^* algebra, Gelfand-Naimark Theorem and its consequences;

CO 4 describe States and Pure states, GNS construction; understand Von-Neumann algebra with examples and applications, Double commutant Theorem.

Detailed Syllabus

- Banach Algebra, Examples and elementary properties, Spectrum and its properties, Spectral radius formula, Maximal ideal space, Gelfand transformation.
- C^*-algebra, Examples and properties, approximate identity, Gelfand-Mazur theorem, Gelfand-Naimark theorem and its applications (Continuous functional calculus), States and pure states, GNS construction.
- Strong and weak operator topology in $\mathcal{B}(H)$, Von-Neumann algebra, projections, double commutant theorem, Kaplansky density theorem, factors.

References

[1] Gerard J. Murphy, *C^*-algebras and operator theory*, Elsevier.

[2] Kehe Zhu, *An introduction to operator algebras*, CRC press.

[3] Bruce Blackadar, *Operator algebras: Theory of C^*-algebras and Von-Neumann algebras*, Springer.

[4] K. R. Davidson, *C^*-algebras by example*, Fields Institute Monographs.

[5] E. C. Lance, *Hilbert C^*-modules*, London Mathematical Society.

[6] Richard V. Kadison, John R. Ringrose, *Fundamentals of the Theory of Operator Algebras, Vol. I and II*, AMS.

Geometry - II (Lie groups, Lie algebras, and Symmetric Spaces)

Semester : IV	Course Type : T
Course ID : MATH1003A2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 know Lie groups, Lie algebras with examples, their fundamental properties;

CO 2 understand solvable and nilpotent Lie algebras, theorems of Lie and Engel;

CO 3 explain fundamental concepts of semisimple and simple Lie algebras such as Cartan subalgebras, root space decompositions, Dynkin diagrams; know classification of complex simple Lie algebras;

CO 4 know Riemannian symmetric spaces, irreducible Riemannian globally symmetric spaces and their classification; understand Hermitian symmetric spaces with examples.

Detailed Syllabus

- Lie groups with examples, Lie algebras with examples, Lie algebra of a Lie group, Lie group homomorphisms and its properties, Lie subgroups, one-one correspondence between connected Lie subgroups of a Lie group and Lie subalgebras of the corresponding Lie algebra, closed Lie subgroups, simply connected Lie groups, exponential map and its properties, adjoint homomorphism and its properties, automorphism group of a Lie algebra as a Lie group, homogeneous manifolds with examples, compact Lie algebras.
- Solvable and nilpotent Lie algebras, theorem of Lie and Engel, Killing form of a Lie algebra, semisimple and simple Lie algebras, Cartan subalgebra of a semisimple Lie algebra, root space decomposition, real forms of complex Lie algebras, compact real form, Cartan decomposition of a real semisimple Lie algebra, classical complex Lie algebras.
- Riemannian locally and globally symmetric spaces, group of isometries of Riemannian globally symmetric spaces, Riemannian symmetric pairs and associated Riemannian globally symmetric spaces, orthogonal symmetric Lie algebras, compact connected Lie groups as Riemannian globally symmetric spaces, totally geodesic submanifolds and Lie triple systems.
- Effective orthogonal symmetric Lie algebras of the compact type, noncompact type and Euclidean type; dual of an orthogonal symmetric Lie algebra; irreducible orthogonal symmetric Lie algebras of type I, II, III, and IV; decomposition of an effective orthogonal symmetric Lie algebra into irreducibles; irreducible Riemannian globally symmetric spaces; decomposition of a simply connected Riemannian globally symmetric space into irreducibles.
- Symmetric spaces of the noncompact type and compact type, maximal compact subgroups of connected semisimple Lie groups, restricted roots, the Iwasawa decomposition of a real semisimple Lie algebra and of a connected semisimple Lie group, Hermitian symmetric spaces, bounded symmetric domains as Hermitian symmetric spaces of the noncompact type.
- Simple complex Lie algebras; Dynkin diagrams; exceptional Lie algebras of type $\mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8, \mathfrak{f}_4, \mathfrak{g}_2$; description of finite order automorphisms of a complex simple Lie algebra, classification of irreducible Riemannian globally symmetric spaces of type I, II, III, and IV.

References

[1] F. W. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, Springer.

[2] S. Helgason, *Differential Geometry, Lie Groups, and Symmetric Spaces*, AMS.

[3] A. Borel, *Semisimple Groups and Riemannian Symmetric Spaces*, TRIM- HBA.

[4] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer.

Abstract Harmonic Analysis

Semester : IV	Course Type : T
Course ID : MATH1003A3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand Banach algebras, ideals and maximal ideals of a Gelfand algebra, maximal ideal space with Gelfand topology, Gelfand representation theorem, theory of non-unital Banach algebras;

CO 2 understand the basics of topological groups, some special locally compact abelian groups;

CO 3 understand the measure theory on locally compact Hausdorff spaces, positive Borel measure, Riesz representation theorem, complex measure Radon-Nikodym theorem and its consequences, bounded linear functionals on $L_p(1 \leq p \leq \infty)$, the dual space of $C_0(X)$ for a locally compact Hausdorff space X (the Riesz representation theorem);

CO 4 understand Haar measure on locally compact groups, its construction, properties and uniqueness (up to multiplicative constant); understand the basic representation theory with examples, unitary representations, Schur's lemma, representations of a group and its group algebra, Gelfand-Raikov theorem and its applications.

Detailed Syllabus

- **Banach Algebra:** Normed algebra, Banach algebra, examples of Banach algebra, resolvent function and its analyticity, spectrum of a point, spectral radius, ideal and maximal ideal of a Gelfand algebra, character space, maximal ideal space with Gelfand topology, Gelfand representation theorem, theory of non-unital Banach algebras.
- **Topological Group:** Basic definition and facts, subgroups, quotient groups, some special locally compact Abelian groups.
- **Measure Theory on Locally Compact Hausdörff Space:** Positive Borel measure, Riesz representation theorem, Complex measure Radon-Nikodym theorem and its consequences, bounded linear functionals on $L_p(1 \leq p \leq \infty)$, the dual space of $C_0(X)$ for a locally compact Hausdörff space X (the Riesz representation theorem).
- **Haar Measure on Locally Compact Group:** Construction of Haar measure, properties of Haar measure, uniqueness of Haar measure (up to multiplicative constant).
- **Basic Representation Theory:** Unitary representations, Schur's lemma, representations of a group and its group algebra, Gelfand-Raikov theorem.

References

[1] Hewitt and Ross, *Abstract Harmonic analysis, Vol. I and II*, Springer-Verlag.

[2] G. B. Folland, *A Course in Abstract Harmonic Analysis*, CRC Press (1995).

[3] L. H. Loomis, *An Introduction to Abstract Harmonic Analysis*, D. Van Nostrand Company Inc.

[4] Bachman and Narici,*Elements of Abstract Harmonic Analysis*, Academic Press, New York.

[5] Y. Katznelson, *An Introduction to Harmonic Analysis*, Dover Publications, Inc.

General Theory of Relativity and Cosmology

Semester : IV	Course Type : T
Course ID : MATH1003B1	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand why Einstein introduced General Theory of Relativity and its experimental tests;

CO 2 know that the geometry of the space-time is connected with the distribution of matter in the universe; understand the physical nature of the Einstein's gravitational equations;

CO 3 understand the structure of our universe in very large scales and they will be able to derive a metric for describing such geometrical structure;

CO 4 obtain a basic knowledge on cosmology and gain some experience on the application of astronomical data sets.

Detailed Syllabus

- Einstein's motivations for general relativity, the principle of equivalence, the principle of general covariance, gravity as geometry, the metric and metric tensor, Riemann curvature tensor, Bianchi identity, Ricci tensor, Einstein tensor, Weyl tensor, geodesics, Einstein's field equations, cosmological constant; Schwarzschild solution, Birkhoff's theorem, experimental tests of general relativity, introduction to black holes and their geometries.
- The cosmological principle, Friedmann-Lemîatre-Robertson-Walker (FLRW) metric, cosmic dynamics, Friedmann equations, equation of state, evolution of the scale factor, big bang theory, early universe, present accelerating expansion of the universe, dark energy - the biggest mystery, cosmological parameters.
- Introduction to various observational datasets, simple numerical codes (in Mathematica/C/Python) and their role in general relativity and cosmology.

References

[1] W. Rindler, *Relativity - special, general, and cosmological*, Oxford University Press.

[2] S. M. Carroll, *Space-time and Geometry*, Addison Wesley.

[3] Ray D'Inverno, *Introducing Einstein's Relativity*, Clarendon Press.

[4] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity*, John Wiley & Sons.

[5] Misner, Thorne and Wheeler, *Gravitation*, W. H. Freeman and Company.

[6] S. Weinberg, *Cosmology*, Oxford University Press.

[7] Ta-Pei Cheng, *Relativity, Gravitation and Cosmology*, Oxford University Press.

[8] B. F. Schutz, *A first course in General Relativity*, Cambridge University Press.

[9] J. B. Hartle, *Gravity, an introduction to Einstein's General Relativity*, Addison Wesley.

[10] A. K. Raychaudhuri, *Theoretical Cosmology*, Oxford University Press.

[11] B. Ryden, *Introduction to Cosmology*, Addison Wesley.

Qualitative Theory of Planar Vector Fields - II

Semester : IV	Course Type : T
Course ID : MATH1003B2	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of the course, a student will be able to:

CO 1 compute Poincaré-Lyapunov constants and illustrate the classical center focus problem to distinguish between a center and a focus;

CO 2 demonstrate an understanding of the Piexoto's theorem to characterize structurally stable vector fields on two dimensional compact manifolds;

CO 3 investigate bifurcation theory of vector fields; derive universal unfolding of codimension one and codimension two local bifurcations;

CO 4 demonstrate an understanding of the cyclicity problem for limit periodic sets, 2nd part of the Hilbert's 16th problem and the weak form of the Hilbert's 16th problem.

Detailed Syllabus

- The Center and Focus Problem: The Classical Poincaré Center-Focus Problem, Lyapunov Numbers, Normal Forms, The Center Variety.
- Isochronus Centers and Linearization: The Period Function, Isochronous Center, Darboux Linearization.
- Structural Stability: Piexoto's Theorem, Structural Stability of Vector Fields on Open Surfaces.
- Bifurcation Theory: Unfolding Vector Fields, Universal Unfolding, Local Codimension 1 and 2 Bifurcations of Singularities, Andrnov-Hopf Bifurcation, Bifurcation of Limit Cycles, Homoclinic Bifurcations, Melnikov Theory, Equivariant Bifurcations.
- The Cyclicity Problem: Limit Periodic Sets, The Cyclicity for Limit Periodic Sets, The Second Part of Hilbert's 16th Problem, The Finite Cyclicity Conjecture, The Weak Form of Hilbert's 16th Problem.

References

[1] F. Dumortier, J. Llibre and J.C. Artés, *Qualitative Theory of Planar Differential Systems*, Springer.

[2] L. Perko, *Differential Equations and Dynamical Systems*, Springer.

[3] V. I. Arnold, *Geometrical Methods in the Theory of Ordinary Differential Equations*, Springer.

[4] V. G. Romanovski and D.S. Shafer, *The Center and Cyclicity Problems: A Computational Algebra Approach*, Birkhauser.

[5] S. N. Chow, C. Z. Li and D. Wang, *Normal Forms and Bifurcation of Planar Vector Fields*, Cambridge University Press.

[6] M. Golubitsky and I. Stewart, *The Symmetry Perspective*, Birkhauser.

[7] R. Roussarie, *Bifurcations of Planar Vector Fields and Hilbert's Sixteenth Problem*, Birkhauser.

Advanced Operations Research - II

Semester : IV	Course Type : T
Course ID : MATH1003B3	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 efficiently handle stochastic inventory models and implement it in practical problems;

CO 2 formulate and solve network problems arising in real life using PERT/CPM Techniques;

CO 3 analyse and solve real life replacement problems.

CO 4 apply learned techniques in agricultural planning, biotechnology, data analysis, distribution of goods and resources, emergency and rescue operations, engineering etc.

Detailed Syllabus

- **Probabilistic Inventory control Models:** Probabilistic demand models. Expected cost. Probabilistic order level systems. Probabilistic order level systems with instantaneous demand. Probabilistic order level systems with uniform demand. Probabilistic order level systems with lead time. Discrete and continuous probability versions of the models. Problems on the two versions of the models. Newspaper boy problem. Spare parts problem. Baking company problem. Equivalence of probabilistic order level systems.
- **Project Scheduling and Network Analysis:** Types of network problems with examples, flows in network, Max-flow min-cut theorem and its application, Introduction and Basic differences between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM network Components and Precedence Relationships, Fulkerson's 'i-j' rule, Critical Path analysis, Forward and backward pass methods, Floats of an activity, Project costs by CPM, Probability in PERT analysis, Project Crashing, Time cost Trade-off procedure, Updating of the Project, Resource Allocation.
- **Replacement Models:** Replacement problem, Types of replacement problems, Replacement of capital equipment that varies with time, Replacement policy for items where maintenance cost increases with time and money value is not considered, Money value, Present worth factor, Discount rate, Replacement policy for item whose maintenance cost increases with time and money value changes at a constant rate, Choice of best machine, Replacement of low cost items, Group replacement, Individual replacement policy, Mortality theorem, Recruitment and promotional problems.

References

[1] John A. Muckstadt, Amar Sapra, *Principles of Inventory Management*, Springer.

[2] Sven Axsater, *Inventory Control*, Springer.

[3] Eliczer Nadder, *Inventory Systems*, John Wiley and Sons.

[4] G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Prentice Hall.

[5] R. J. Tersine and M. Hays, *Principles of Inventory and Material Management*, Pearson.

[6] A. Ravindran, Don T. Phillips, James J. Solberg, *Operations Research: Principles and Practice*, Wiley.

[7] H. S. Taha, *Operations Research*, Pearson Education.

[8] Frederick S. Hillier, Gerald J. Lieberman, *Introduction to Operations Research*, McGraw Hill Education.

[9] Sasieni Maurice, Arther Yaspan, Lawrence Friedman, *OR methods and Problems*, Wiley.

[10] S. D. Sharma, *Operations Research*, Kedar Nath.

Mathematical Biology - II

Semester : IV	Course Type : T
Course ID : MATH1003B4	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 gain an idea about the ecological modeling and infectious disease modeling;

CO 2 understand the concept of basic reproduction number R_0;

CO 3 understand the basic concept of reaction-diffusion equations;

CO 4 apply creatively the knowledge of Mathematics in selected real life situations.

Detailed Syllabus

- Difference Equation and its Application: Difference Calculus, Linear first – order difference equations, Nonlinear difference equations, Higher order linear difference equations, Systems of difference equations, Stability Theory, Applications.
- An introduction to partial differential equations and diffusion in biology: Functions of several variables: a review; Random motion and diffusion equation; Diffusion equations and some of its consequences
- Partial differential equation models in biology: Population dispersal models based on diffusion, Density-dependent dispersal, Simple solutions: steady states and travelling waves, Homogeneous steady states, Travelling wave solutions.
- Models for development and pattern formation in biological systems: Homogeneous steady states and inhomogeneous perturbations, conditions for diffusion instability, Physical explanation, Extension to higher dimensions and finite domain.

References

[1] J. D. Murray, *Mathematical Biology - II*, Springer and Verlag.

[2] Leach Edelstein-Keshet, *Mathematical Models in Biology*, The Random House/ Birkhauser Mathematics Series.

[3] L. Perko, *Differential Equations and Dynamical Systems*, Springer Verlag.

[4] D. W. Jordan and P. Smith, *Nonlinear Ordinary Equations- An Introduction to Dynamical Systems*, (Third Edition), Oxford University Press.

[5] S. Goldberg, *Introduction to Difference Equations with illustrative examples from Economics, Psychology and Sociology*, 1987, Dover Books on Mathematics.

Advanced Numerical Analysis - II

Semester : IV	Course Type : T
Course ID : MATH1003B5	Full Marks : 50

Course Structure

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 acquaint the students with basics of numerical integration by quadrature; compare and contrast the pros and cons of various quadrature methods like Newton-Cotes, Gaussian Quadrature and Extrapolation methods;

CO 2 acquaint the students with the methods for numerically solving differential equations; understand the differences between single step and multi-step methods and their relative accuracy and utility;

CO 3 acquaint the students with various finite difference methods for solving partial differential equations numerically; know the CFL conditions and their use;

CO 4 apply the above numerical methods using the C programming language.

Detailed Syllabus

- Integration: Newton-Cotes method, derivation of Trapezoidal, Simpson's $\frac{1}{3}$ rule and Simpson's $\frac{3}{8}$ rule using N-C formula, Gaussian Quadrature method, Richardson extrapolation.
- Ordinary differential equations: Recapitulation of Runge Kutta methods, Multistep methods: Adams methods (Adam Bashforth, Adam Moulton), Picard's method.
- Partial differential equations: Introduction to finite difference methods:
  1. Heat equation: Explicit finite difference scheme, implicit Crank-Nicholson scheme, errors and stability.
  2. Wave equation and Poisson equations: Forward schemes, Leapfrog method, Lax-Friedrichs method, Lax-Wendroff method, stability analysis, Von-Neumann analysis, the CFL conditions.
- Computer programming and code development using C/Python for Gaussian quadrature, Adam's multistep methods, solving Laplace and Poisson equation by finite difference methods and visualization of graphical results wherever possible.

References

[1] K. Atkinson, *Introduction to Numerical Analysis*, J. Wiley and Sons.

[2] E. Isaacson and H. B. Keller, *Analysis of Numerical Methods*, Dover Publications.

[3] F. B. Hildebrand, *Introduction to Numerical Analysis*, Dover Publications.

[4] J. Stoer, R. Bulirsch, *Introduction to Numerical Analysis*, Springer Science.

[5] W.Cheney, D. Kincaid, *Numerical Mathematics and Computing*, Brooks/Cole.

[6] William H. Press, Brian P. Flannery, Saul Teukolsky, William T. Vetterling, *Numerical Recipes in C: The Art of Scientific Computing*, Cambridge University Press.