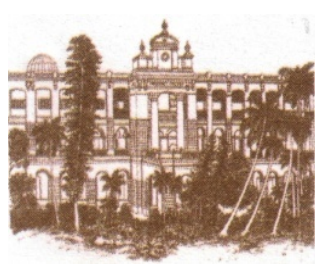
PRESIDENCY UNIVERSITY

DEPARTMENT OF MATHEMATICS

Syllabus for B.Sc. (Hons.) Mathematics
(under the Choice Based Credit System as proposed by UGC)
Approved by the Board of Studies (BoS) of the Department of Mathematics
(with effect from 2022-23)



Department of Mathematics
(Faculty of Natural and Mathematical Sciences)
Presidency University, Kolkata
Hindoo College (1817-1855), Presidency College (1855-2010)
86/1, College Street, Kolkata - 700 073
West Bengal, India

Programme Outcomes

PO 1 Developing Analytical and Real-Life Skills: Students will be able to know the importance of mathematical modelling, simulation and computational methods to solve real world problems. They will be able to model physical, biological, environmental, statistical etc. problems using mathematical knowledge. They will be able to analyse and suggest acceptable real- life solutions using mathematical and data interpretation skills.

PO 2 Promoting Higher Education: Students completing this programme will be able to apply their knowledge in Mathematics to construct and develop logical arguments for the solution of complex mathematical problems, describe and formulate mathematical ideas from multiple perspectives. They will be able to explain and apply fundamental concepts of mathematics for solving advanced research problems.

PO 3 Enhancing Employability in Industry: Students will be able to use the knowledge acquired in related areas of computer science, statistics and Programming Languages to enhance their employability for government jobs, jobs in software engineering, data science, banking, insurance and investment sectors and in various other public and private enterprises.

PO 4 Inculcating Innovation and Creativity: Students will be able to undertake independent research initiatives in mathematics. They will be able to create and hypothesise mathematical results. Will be able to estimate and understand and analyse the limitations of a method and suggest appropriate remedies for tackling such problems.

Department of Mathematics
Presidency University
Course Structure for B.Sc. (Hons.) Mathematics under CBCS

Semester	Core Course (14)	Ability Enhancement Compulsory Course (AECC) (2)	Skill Enhancement Course (SEC) (2)	Discipline Specific Elective (DSE) (4)	General Elective (GE) (4)
I	**C1** – Calculus and Geometry (P)	**AECC-1** English Language			**GE-1**
	C2 – Algebra				
II	**C3** – Real Analysis I	**AECC-2** Environmental Science			**GE-2**
	C4 – Groups and Rings I				
III	**C5** – Real Analysis II		Computer Programming with C (**SEC-1A**)/ Computer Programming with Python (**SEC-1B**) (P)		**GE-3**
	C6 – Linear Algebra I				
	C7 – Ordinary Differential Equations (P)				
IV	**C8** – Sequence and Series of Functions and Metric Spaces		**SEC-2** LaTeX (P)		**GE-4**
	C9 – Multivariate Calculus				
	C10 – Partial Differential Equations (P)				
V	**C11** – Numerical Methods (P)			**DSE-1**	
	C12 – Groups and Rings II			**DSE-2**	
VI	**C13** – Complex Analysis and Fourier Series			**DSE-3**	
	C14 – Probability Theory			**DSE-4**	

∗ Here the alphabet 'P' present in some course denotes that practical is one of the components of that course.

Details of the Credit Structure of Courses:

Sl. No.	Course	Credit	Theory	Tutorial/Practical	Marks*
1	Core Course	6×14	$5 \times 13 + 4 \times 1$	$1 \times 13 + 2 \times 1$	$100 \times 14 = 1400$
2	DSE	6×4	5×4	1×4	$100 \times 4 = 400$
3	GE	6×4	5×4	1×4	$100 \times 4 = 400$
4	SEC	4×2	4×2		$100 \times 2 = 200$
5	AECC	4×2	4×2		$100 \times 2 = 200$
	Total	**148**	**125**	**23**	**2600**

$*$ For all the core courses, division of marks would be $100 = 70$ (theory exam.) + 30 (internal assessment) while for the General Electives (GE), the division of marks would be $100 = 80$ (theory exam.) + 20 (internal assessment).

Discipline Specific Electives (DSE):

1. **Choices for DSE-1**
   (a) Linear Programming and Game Theory
   (b) Discrete Mathematics and Number Theory (P)
2. **Choices for DSE-2**
   (a) Theory of Ordinary Differential Equations (ODE)
   (b) Mathematical Modelling (P)
3. **Choices for DSE-3**
   (a) Linear Algebra II and Field Theory
   (b) A Mathematical Primer to Data Science (P)
4. **Choices for DSE-4**
   (a) Mechanics (P)
   (b) Differential Geometry

Generic Electives (to be offered to the students of other departments) (GE):

1. **GE-1:** Differential Calculus
2. **GE-2:** Integral Calculus and Differential equations
3. **GE-3:** Algebra - I
4. **GE-4:** Algebra - II

Detailed Syllabi of the Courses

Core 1: Calculus and Geometry
Subject Code: MATH 01C1
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply mean value theorems in determining concavity, points of inflections of smooth functions; explain asymptotes, cusps, vertical tangents; trace various functions such as polynomials, rational functions, exponential functions, sine and cosine functions;

CO 2 identify and trace conics represented by general second degree equations in two variables; illustrate the theory of planes, straight lines, spheres, cones, cylinders, surface of revolutions, conicoids in three dimensions;

CO 3 determine the length of smooth curves, area of surface of revolutions, volume of solid of revolutions;

CO 4 compute curvature and torsion of smooth curves in three dimensions.

Detailed Syllabus

Module 1: Higher order derivatives, Leibniz rule and its applications to problems of type $e^{ax+b}\sin x$, $e^{ax+b}\cos x$, $(ax+b)^n\sin x$, $(ax+b)^n\cos x$, concavity and inflection points, asymptotes in cartesian and polar coordinates, curve tracing in cartesian and polar coordinates, L'Hôpital's rule, applications in business, economics and life sciences. Visualisation using any software.

Module 2: Reduction formulae, derivations and illustrations of reduction formulae of the type $\int \sin nx dx$, $\int \cos nx dx$, $\int \tan nx dx$, $\int \sec nx dx$, $\int (\log x)^n dx$, $\int \sin^n x \sin^m x dx$, surface of revolution, volumes by slicing, disks and washers methods, volumes by cylindrical shells, parametric equations, parameterizing a curve, arc length, arc length of parametric curves. Visualisation using any software.

Module 3: Triple product, introduction to vector functions, operations with vector-valued functions, limits and continuity of vector functions, differentiation and integration of vector functions, tangent and normal components of acceleration, modeling ballistics and planetary motion, Kepler's second law.

Module 4: Classification of conics using the discriminant in two dimensional geometry. Equation of a plane, signed distance of a point from a plane, planes passing through the intersection of two planes, angle between two intersecting planes and their bisectors. Parallelism and perpendicularity of two planes. Equations of a line in space, rays or half lines, direction cosines of a line, angle between two lines, distance of a point from a line, condition for coplanarity of two lines, skew-lines, shortest distance. Spheres, tangent plane of a sphere, orthogonal spheres, cylindrical surfaces, cone, introduction to conicoids.

Books Recommended:

1. G. B. Thomas and R. L. Finney, Calculus, Pearson.
2. M. J. Strauss, G. L. Bradley and K. J. Smith, Calculus, Pearson.
3. H. Anton, I. Bivens and S. Davis, Calculus, John Wiley.
4. R. Courant and F. John, Introduction to Calculus and Analysis I & II, Springer.
5. T. M. Apostol, Calculus I & II, John Wiley.
6. S. L. Loney, The Elements of Coordinate Geometry, McMillan.
7. R. J. T. Bell, Elementary Treatise on Coordinate Geometry of Three Dimensions, McMillan.

Core 2: Algebra
Subject Code: MATH 01C2
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 use De Moivre's theorem in a number of applications to solve numerical problems; determine roots of real and complex polynomials using various methods;

CO 2 explain relations, equivalence relations and partitions;

CO 3 recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix using rank, and solve consistent system of linear equations; find eigenvalues and corresponding eigenvectors for a square matrix;

CO 4 get a preliminary idea about groups and rings.

Detailed Syllabus

Module 1: Quick review of algebra of complex numbers, modulus and amplitude (principal and general values) of a complex number, polar representation, De-Moivre's Theorem and its applications: nth roots of unity.

Module 2: Polynomials with real coefficients and their graphical representation. Relationship between roots and coefficients: Descarte's rule of signs, symmetric functions of the roots, transformation of equations. Solutions of the cubic and bi-quadratic equations by Cardan's and Ferrai's methods. Statement of the fundamental theorem of Algebra. Inequality: The inequality involving AM$\geq$GM$\geq$HM, Cauchy-Schwartz inequality.

Module 3: Set, Power Set, Equivalence relations and partitions, partial order, statement of Zorn's lemma. Mappings and functions, injective, surjective and bijective mappings, composition of mappings, invertible mappings. Cardinality of a set, countable and uncountable sets, bijection from the unit interval to unit square using Schroeder-Bernstein theorem, well ordering principle. Divisibility and Euclidean algorithm, congruence relation between integers. Principle of strong and weak mathematical induction and relationship between them. Statement of the fundamental theorem of arithmetic.

Module 4: Elementary row operations: row reductions, elementary matrices, echelon forms of a matrix, rank of a matrix, characterization of invertible matrices using rank. Solution of systems of linear equations $A\mathbf{x} = \mathbf{b}$: Gaussian elimination method and matrix inversion method.

Module 5: Elements of $\mathbb{R}^n$ as vectors, linear combination and span of vectors in $\mathbb{R}^n$, linear independence and basis, vector subspaces of $\mathbb{R}^n$, dimension of subspaces of $\mathbb{R}^n$. Linear transformations on $\mathbb{R}^n$ as structure preserving maps, invertible linear transformations, matrix of a linear transformation, change of basis matrix. Adjoint, determinant and inverse of a matrix.

Module 6: Definitions and examples: (i) groups, subgroups, cosets, normal subgroups, homomorphisms. (ii) rings, subrings, integral domains, fields, characteristic of a ring, ideals, ring homomorphisms.

Books Recommended:

1. S. Bernard and J. M. Child, Higher Algebra, Macmillan.
2. S. K. Mapa, Classical Algebra, Levant.
3. T. Andreescu and D. Andrica, Complex Numbers from A to Z, Birkhauser.
4. D. C. Lay, Linear Algebra and its Applications, Pearson.
5. C. Curtis, Linear Algebra, Springer.
6. J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.

Core 3: Real Analysis I
Subject Code: MATH 02C3
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 get an idea about the real number system and its algebraic, order properties; explain important properties of $\mathbb{R}$ and its subsets.

CO 2 explain the concept of sequence of real numbers and its convergence, different criteria of a sequence which makes it a convergent sequence, Cauchy sequence, contractive sequence, subsequence, limit superior and limit inferior etc;

CO 3 elucidate the concept of infinite series of real numbers, its convergence and different tests to check the convergence of a given series;

CO 4 implement the limit and continuity of a function at a point and its geometry, sequential criterion of limit and divergence criterion, various results for the existence of the limit of a function at a point, infinite limits and limits at infinity; uniform continuity, non-uniform continuity criteria, the continuous extension theorem in practical problems.

Detailed Syllabus

Module 1: Review of Algebraic and Order Properties of $\mathbb{R}$, δ-neighbourhood of a point in $\mathbb{R}$, countability of $\mathbb{Q}$ and uncountability of $\mathbb{R}$. Bounded above sets, Bounded below sets, Bounded Sets, Unbounded sets, Supremum and Infimum, The Completeness Property of $\mathbb{R}$, The Archimedean Property, Density of Rational (and Irrational) numbers in $\mathbb{R}$, Intervals. Limit points of a set, Isolated points, Open sets, Closed sets, Illustrations of Bolzano-Weierstrass theorem for sets.

Module 2: Sequences, Bounded sequence, Convergent sequence, Limit of a sequence, Limit superior and Limit inferior, Limit Theorems, Monotone Sequences, Monotone Convergence Theorem. Subsequences, Divergence Criteria, Monotone Subsequence Theorem (statement only), Bolzano-Weierstrass Theorem for Sequences. Cauchy sequence, CauchyÕs Convergence Criterion. Contractive sequence, Construction of $\mathbb{R}$ from $\mathbb{Q}$ by Dedekind's cut or by equivalent Cauchy sequences.

Module 3: Infinite series, convergence and divergence of infinite series, Cauchy Criterion, Tests for convergence: Comparison test, Limit Comparison test, D'Alembert's Ratio Test, Cauchy's root test, integral test, Alternating series, Leibniz test, Condensation test, Raabe's test, Absolute and Conditional convergence, Cauchy product, Rearrangements of terms, Riemann's theorem on rearrangement of series (statement only).

Module 4: Limits of functions (ϵ-δ approach), sequential criterion for limits, divergence criteria. Limit theorems, Squeez theorem, one sided limits. Infinite limits and limits at infinity.

Module 5: Continuous functions, sequential criterion for continuity and discontinuity. Algebra of continuous functions. Continuous functions on an interval, intermediate value theorem, location of roots theorem, preservation of intervals theorem. Uniform continuity, non-uniform continuity criteria, uniform continuity theorem. Lipscitz function. The continuous extension theorem.

Books Recommended:

1. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, Inc.
2. T. Tao, Analysis I, HBA.
3. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
4. T. M. Apostol, Mathematical Analysis, Narosa.
5. S. K. Berberian, A First Course in Real Analysis, Springer Verlag

Core 4: Groups & Rings I
Subject Code: MATH 02C4
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 link the fundamental concept of groups and symmetries of geometrical objects; explain fundamental properties of permutation groups and alternating groups; elucidate the basic concepts in group theory and applications of Lagrange's theorem; understand free groups;

CO 2 interpret the fundamental concepts in ring theory such as the concepts of ideals, prime ideals, maximal ideals, quotient rings, integral domains etc;

CO 3 illustrate Chinese remainder theorem and its applications;

CO 4 apply above concepts in various fields like Physics, Statistics, Computer Science, Chemistry.

Detailed Syllabus

Module 1: Groups: Groups as symmetries, examples: S_n, A_n, $\mathbb{Z}_n$, D_n, $GL(n,\mathbb{R})$, $SL(n,\mathbb{R})$, $O(n,\mathbb{R})$, $SO(n,\mathbb{R})$ etc., elementary properties of groups, abelian groups.

Module 2: Subgroups and cosets: examples of subgroups including centralizer, normalizer and center of a group, product subgroups; cosets and Lagrange's theorem with applications. Cyclic groups and its properties, classification of subgroups of cyclic groups. Normal subgroups: properties and examples, conjugacy classes of elements, properties of homomorphisms and kernels, quotient groups and their examples. Presentation of a group in terms of generators and relations. Free groups and its universal properties.

Module 3: Properties of S_n, cycle notation for permutations, even and odd permutations, cycle decompositions of permutations in S_n, alternating group A_n, Cayley's theorem. Isomorphism theorems: proofs and applications, isomorphism classes of finite groups of lower order. Cauchy's theorem for finite abelian groups, statements of Cauchy's and Sylow's theorem and applications.

Module 4: Rings: Examples and basic properties of rings, subrings and integral domains. Ideals, algebra of ideals, quotient rings, chinese remainder theorem.

Module 5: Prime and maximal ideals, quotient of rings by prime and maximal ideals, ring homomorphisms and their properties, isomorphism theorems, field of fractions.

Module 6: Applications of Algebra in various fields.

Books Recommended:

1. I. N. Herstein, Topics in Algebra, John Wiley.
2. J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.
3. M. Artin, Algebra, Pearson.
4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, Cambridge Univ. Press.
5. J. A. Gallian, Contemporary Abstract Algebra, Narosa.
6. M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.
7. J. J. Rotman, An Introduction to the Theory of Groups, Springer.
8. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley.
9. T. W. Hungerford, Algebra, Springer.
10. M. K. Sen, S. Ghosh, P. Mukhopadhyay and S. K. Maity, Topics in Abstract Algebra, University Press.

Core 5: Real Analysis II
Subject Code: MATH 03C5
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply the concept of differentiability of real valued functions and to understand various consequences of mean value theorems of differential calculus;

CO 2 approximate real life phenomena using Taylor's theorem, Taylor's series and radius of convergence;

CO 3 illustrate Riemann integrability of bounded functions and algebra of Riemann integrable functions; use the fundamental theorems and mean value theorems of integral calculus in various real world problems; implement the concepts of improper integrals and use various tests for convergence of improper integrals and series;

CO 4 apply the properties of Beta and Gamma functions in various branches of science; identify functions of bounded variations and their properties;

Detailed Syllabus

Module 1: Differentiability of a function at a point and in an interval, Carathéodory's theorem, Algebra of differentiable functions, Chain rule, Inverse functions, Relative extrema, interior extremum theorem. Rolle's theorem, Mean value theorem, intermediate value property of derivatives - Darboux's theorem. Cauchy's mean value theorem. Applications of mean value theorems to inequalities and approximation of polynomials. Proof of L'Hôpital's rule.

Module 2: Taylor's theorem with Lagrange's form of remainder and Cauchy's form of remainder, application of Taylor's theorem to convex functions. Taylor's series and Maclaurin's series expansions of exponential, trigonometric and other functions. Radius of convergence, Cauchy-Hadamard theorem.

Module 3: Riemann integration. upper and lower sums, Riemann's conditions of integrability. Riemann sum and definition of Riemann integral through Riemann sums. Equivalence of the two definitions. Riemann integrability of monotone and continuous and piecewise continuous functions. Properties of the Riemann integral. Intermediate Value theorem for integrals. Fundamental theorems of Calculus and its consequences. Substitution theorem. Functions of bounded variation and their properties.

Module 4: Improper integrals; Proof of integral test for series, Convergence of Beta and Gamma functions, Abel's test, Dirichlet's test, Bohr-Mollerup theorem and its consequences.

Books Recommended:

1. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley.
2. T. Tao, Analysis I & II, HBA.
3. T. M. Apostol, Mathematical Analysis, Narosa.
4. T. M. Apolstol, Calculus I, Wiley.
5. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
6. C. G. Denlinger, Elements of Real Analysis, Jones & Bartlett.
7. K. A. Ross, Elementary Analysis, The Theory of Calculus, Springer (UTM).
8. H. L. Royden, Real Analysis, Pearson.

Core 6: Linear Algebra I
Subject Code: MATH 03C6
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply fundamental concepts of vector spaces and linear transformations;

CO 2 determine the rank of a matrix;

CO 3 check the diagonalizability of matrices and linear transformations; determine Rational canonical form and Jordan form of matrices and linear transformations;

CO 4 interpret the geometry of inner product spaces; understand the concept of orthogonal diagonalizability of real symmetric matrices.

Detailed Syllabus

Module 1: Vector spaces, subspaces, algebra of subspaces, linear combination of vectors, linear span, linear independence, basis and dimension, dimension of subspaces, quotient spaces.

Module 2: Linear transformations, null spaces and ranges, rank and nullity of a linear transformation, Sylvester's (rank-nullity) theorem and application, matrix representation of a linear transformation, Algebra of linear transformations, Invertibility and Isomorphisms, change of coordinate matrix.

Module 3: Row space and column space of a matrix, row rank, column rank and rank of a matrix, equality of these ranks, rank of product of two matrices, rank factorisation.

Module 4: Dual Spaces, dual basis, double dual, transpose of a linear transform and its matrix in the dual basis, annihilators.

Module 5: Eigen values and eigen vectors, eigen space of a linear transform, diagonalizability of a matrix, invariant subspaces and Cayley-Hamilton theorem, minimal polynomial for a linear operator, diagonalizability in connection with minimal polynomial, canonical forms (Jordan and Rational).

Module 6: Inner product spaces and norms, Cauchy-Schwarz inequality, orthogonal and orthonormal basis, orthogonal projections, Gram-Schmidt orthogonalisation process and orthogonal complements, Bessel's inequality, the adjoint of a linear operator and its basic properties.

Module 7: Definitions of real symmetric, orthogonal, Hermitian, normal, unitary matrices; spectral theorems for real symmetric matrices.

Books Recommended:

1. K. Hoffman and R. A. Kunze, Linear Algebra, PHI.
2. S. Lang, Introduction to Linear Algebra, Springer.
3. G. Strang, Linear Algebra and its Applications, Academic.
4. S. Kumaresan, Linear Algebra: A Geometric Approach, PHI.
5. P. R. Halmos, Finite Dimensional Vector Spaces, Springer.
6. A. R. Rao and P. Bhimasankaram, Linear Algebra, HBA.
7. S. H. Friedberg, A. Insel and L. E. Spence, Linear Algebra, Pearson.
8. C. Curtis, Linear Algebra: An Introductory Approach, Springer (UTM).

Core 7: Ordinary Differential Equations
Subject Code: MATH 03C7
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand how differential equations arise in real life problems; determine the existence and uniqueness of various initial value problems;

CO 2 solve homogeneous and inhomogeneous linear differential equations using various methods;

CO 3 apply the Sturm-Liouvelli problem to determine the eigenvalues and eigenfunctions of various boundary value problems that appear in various branches of science;

CO 4 apply the power series method or the Frobenious method to solve various differential equations; construct and apply various special functions, e.g. Legendre, Bessel, hypergeometric etc.

Detailed Syllabus

Module 1: Formation of ordinary differential equations, geometric interpretation, general solution, particular solution.

Module 2: First order equations, Existence and Uniqueness condition, Lipschitz condition (Statement only), linear first-order equations, exact equations and integrating factors, separable equations, linear and Bernoulli forms. Higher degree equations: general solution and singular solution, p-discriminant and c-discriminant, envelopes of a family of integral curves.

Module 3: Higher order equations: second order equations, general theory and solution of a homogeneous equation, Wronskian (properties and applications), general solution of a non-homogeneous equation, Euler-Cauchy forms, method of undermined coefficients, normal form, variation of parameters, use of $f(D)$ operator, solution of both homogeneous and inhomogeneous higher order equations (order greater than two).

Module 4: Regular Sturm-Liouville problem, eigenvalues and eigenfunctions.

Module 5: Systems of linear differential equations: basic theory, normal form, homogeneous linear systems with constant coefficients.

Module 6: Power series solution: solution about regular singular point, applications: hypergeometric and Legendre differential equations, properties of both functions.

Module 7: Practicals: Solutions of first order and second order ODE using any software. Solution of systems of ODE using any software. Visualization of solutions of ODE arising in mathematical physics or mathematical biology.

Books Recommended:

1. D. A. Murray, Introductory Course on Differential Equations, Longmans, Green and Co.

2. S. L. Ross, Differential Equations, John Wiley and Sons.

3. H. T. H. Piaggio, An Elementary Treatise On Differential Equations, G.Bell And Sons Limited.

4. G. F. Simmons, Differential Equation with Applications and Historical Notes, CRC Press.

5. A. C. King, J. Billingham and S. R. Otto, Differential Equations: Linear, Nonlinear, Ordinary, Partial, Cambridge University Press.

6. C. H. Edwards and D. E. Penny, Differential equations and boundary value problems: Computing and Modelling, Pearson.

Core 8: Sequence & Series of Functions and Metric Spaces
Subject Code: MATH 04C8
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 apply the concepts of pointwise and uniform convergence of sequences and series of functions and consequences of uniform convergence; apply the concept of power series, termwise integration and differentiation;

CO 2 illustrate several concepts of metric spaces and their properties, e.g. completeness, Bolzano Weierstrass property, compactness, and connectedness;

CO 3 explain the continuity of a function defined on metric spaces and homeomorphism;

CO 4 have ideas about Banach contraction mapping theorem, Arzela-Ascoli theorem on metric spaces and apply in various branches of science.

Detailed Syllabus

Sequence & Series of functions**:**
Module 1: Point-wise and uniform convergence of sequence of functions, Cauchy criterion for uniform convergence, continuity, derivability and integrability of the limit function of a sequence of functions, Series of functions, continuity, derivability and integrability of the sum function, Weierstrass' M-Test, construction of nowhere differentiable continuous maps on $\mathbb{R}$.

Module 2: Power series, radius of convergence, Cauchy-Hadamard Theorem, Differentiation and integration of power series; Abel's Theorem; Weierstrass' Approximation Theorem.

Metric spaces**:**
Module 3: Definitions and examples of metric spaces, Neighbourhood of a point, Interior point and interior of a set, limit point and closure of a set, open set and closed set, bounded sets and diameter of a set, dense subsets, subspaces, equivalent metrics, separable spaces.

Module 4: Convergence of sequences in metric spaces, Cauchy sequences, Completeness, Completion of a metric space, Category properties and Baire Category Theorem.

Module 5: Continuity of functions, sequential criterion of continuity, uniform continuity, homeomorphisms, isometry.

Module 6: Compactness, Heine-Borel theorem in $\mathbb{R}$, total boundedness, sequential compactness, compactness and continuity.

Module 7: Connectedness, examples of connected metric spaces, connected subsets of $\mathbb{R}$, connectedness and continuity.

Module 8: Contraction mappings, Banach contraction principle, C(X) as a metric space. Arzelá-Ascoli Theorem, Stone-Weierstrass Theorem (statement only) (if time permits).

Books Recommended:

1. T. Tao, Analysis II, HBA (TRIM Series).
2. T. M. Apostol, Mathematical Analysis, Narosa.
3. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
4. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill.
5. S. Kumaresan, Topology of Metric Spaces, Narosa.
6. I. Kaplansky, Set Theory and Metric Spaces, AMS Chelsea Publishing.
7. S. Shirali and H. L. Vasudeva, Metric Spaces, Springer.
8. J. Munkres, Topology, Pearson.

Core 9: Multivariate Calculus
Subject Code: MATH 04C9
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concept of limit and continuity for functions of several variables; be familiar with partial derivatives, directional derivatives; apply important results for differentiable functions, e.g. mean value theorem, inverse and implicit function theorems in higher dimensions;

CO 2 apply the higher order derivatives related results, e.g. Taylor's theorem, maxima and minima theorem; apply Lagrange's multipliers in constrained optimization problems;

CO 3 illustrate the notions of the tangent space, vector field and properties of gradient vector field; understand and apply the line integral of a vector field over piecewise smooth curves;

CO 4 conceptualize double integral over rectangular/non-rectangular region, polar coordinates and conceptualize triple integral in cylindrical and spherical coordinates; understand the change of variable in double and triple integrals; apply Green's theorem, Gauss divergence theorem, Stoke's theorem in determining surface and volume integrals.

Detailed Syllabus

Module 1: Functions of several variables, limit and continuity, partial derivatives, differentiability and total derivatives as matrices, sufficient condition for differentiability, chain rule; gradient vector, directional derivatives, mean value theorem and inequality in higher dimensions, inverse and implicit function theorems.

Module 2: Higher derivatives and Taylor's theorem, maxima and minima, rank theorem constrained optimisation problems, Lagrange's multipliers.

Module 3: Tangent spaces, definition of a vector field, divergence and curl of a vector field, identities involving gradient, curl and divergence; maximality and normality properties of the gradient vector field.

Module 4: Double integrals, double integrals: (i) over Rectangular region, (ii) over non-rectangular regions, (iii) in polar co-ordinates; triple integrals, triple integrals over: (i) a parallelepiped, (ii) solid regions; computing volume by triple integrals, in cylindrical and spherical co-ordinates; change of variables in double integrals and triple integrals with proof.

Module 5: Line integrals, applications of line integrals: mass and work, fundamental theorem for line integrals, conservative vector fields, independence of path. Green's theorem, Surface integrals, Stoke's theorem, The Divergence theorem.

Books Recommended:

1. T. Tao, Analysis II, HBA (TRIM Series).
2. T. M. Apostol, Mathematical Analysis, Narosa.
3. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
4. T. M. Apostol, Calculus II, John Wiley.
5. M. Spivak, Calculus on Manifold.
6. Charles C. Pugh, Real Mathematical Analysis, Springer.
7. E. Marsden, A. J. Tromba and A. Weinstein, Basic Multivariable Calculus, Springer.
8. J. Stewart, Multivariable Calculus, Concepts and Contexts; Thomson.

Core 10: Partial Differential Equations
Subject Code: MATH 04C10
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concept of partial differential equations and their existence and uniquenesses;

CO 2 solve the first and second order partial differential equations including heat equations, wave equations and Laplace equations;

CO 3 understand how these equations have immense applications in mathematical physics and applied sciences like engineering and technology;

CO 4 understand quite clearly that the realistic problems in nature do not always appear in terms of ordinary differential equations, and the partial differential equations could give a complete picture of a particular realistic problem.

Detailed Syllabus

Module 1: Partial differential equations: basic concepts and definitions, mathematical problems. first- order equations: classification, construction and geometrical interpretation; method of characteristics for obtaining general solution of quasi linear equations, canonical forms of first-order linear equations. Method of separation of variables for solving first order partial differential equations. Lagrange's method and its geometric interpretations, Charpit's Method.

Module 2: Classification of second order linear equations as hyperbolic, parabolic or elliptic; reduction of second order linear quasilinear equations to canonical forms.

Module 3: The Cauchy problem, the Cauchy-Kovalevskaya theorem, 1 dimensional homogeneous wave equation, heat equation and Laplace equation. Vibrating string with fixed end points, heat equation in a rod, Laplace equation in a rectangle and disc.

Module 4: Review of Fourier Series and Fourier coefficients and orthogonality. Solution of wave, heat and Laplace equation by the method of separation of variables and Fourier Series.

Module 5: Practicals (Using any software): Plotting the integral surfaces and characteristics fora first order PDE. Visualization of solution for the second order canonical PDEs.

Books Recommended:

1. I. N. Sneddon: Elements of Partial Differential equations.

2. E. T. Copson: Partial Differential Equations.

3. T. Amarnath, An elementary course in partial differential equations, Narosa.
4. T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, Springer.

Core 11: Numerical Methods
Subject Code: MATH 05C11
Credits: 6 (4 Theory lectures + 2 Practical sessions per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concepts of various errors in numerical evaluation of Mathematical Problems; apply various methods to interpolate a set of data points;

CO 2 numerically solve transcendental and nonlinear equations, algebraic system of equations, using direct or iterative techniques;

CO 3 numerically solve integrals and ordinary differential equations of first order;

CO 4 implement the above methods using C language.

Detailed Syllabus

Module 1: Errors (Absolute, relative, round off, truncation).

Module 2: Solution of transcendental and non linear equations: bisection method, secant and Regula-Falsi methods, iterative methods, Newton's methods with gradient descent algorithm (steepest descent), convergence and errors of these methods.

Module 3: Interpolation: Lagrange's and Newton's divided difference methods, Newton's forward and backward difference methods, Gregory's forward and backward difference interpolation, error bounds of these methods, Hermite interpolation.

Module 4: Solution of a system of linear algebraic equations: LU decomposition, Gaussian elimination method, Gauss-Jordan, Gauss-Jacobi and Gauss-Siedel methods and their convergence analysis.

Module 5: Numerical integration: trapezoidal rule, Simpson's rule, composite trapezoidal and Simpson's rule, Bolle's rule, midpoint rule, Simpson's 3/8-th rule, error analysis of these methods.

Module 6: Ordinary differential equations: modified Euler's method and Runge-Kutta method of second and fourth orders.

Module 7: List of Practicals (Using any software):

1. Root finding using bisection, Newton-Raphson, secant and Regula-Falsi method.

2. LU decomposition.

3. Gauss-Jacobi method.

4. Gauss-Siedel method.
5. Interpolation using Lagrange's and Newton's divided differences.
6. Integration using Simpson's Rule.
7. Differentiation using Runge-Kutta method.

Books Recommended

1. K. Atkinson, Introduction to Numerical Analysis, John Wiley & Sons.
2. W. H. Press, S. A. Teukolsky, W. T. Vettering and B. P. Flannery, Numerical Recipes in C, Cambridge University Press.
3. R. L. Burden and J. D. Faires, Numerical Analysis, Brooks/Cole.
4. U. Ascher and C. Greif, A First Course in Numerical Methods, PHI.
5. J. Mathews and K. Fink, Numerical Methods using Matlab, Pearson.
6. M. K. Jain, S. R. K. Iyengar and R. K. Jain, Numerical Methods for Scientific & Engineering Computation, New Age International (P) Limited.
7. S. S. Sastry: Introductory Methods of Numerical Analysis, PHI.

Core 12: Groups & Rings II
Subject Code: MATH 05C12
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand group actions and the importance of permutation groups; apply Sylow's theorems in determining groups of a given order;

CO 2 determine groups of automorphisms for finite and infinite cyclic groups; determine all finite abelian groups of a given order;

CO 3 explain some advanced concepts of ring theory such as principal ideal domain, Euclidean domain, unique factorization domain etc;

CO 4 apply Eisenstein's criterion to check irreducibility of polynomials with coefficients in a unique factorization domain.

Detailed Syllabus

Module 1: Groups: Group actions, stabilisers and kernels, orbit-stabiliser theorem and applications, permutation representation associated to a group action, Cayley's theorem via group action, groups acting on themselves by conjugation, class equation and consequences, conjugacy in S_n, p-groups, proof of Cauchy's theorem and Sylow's theorems and consequences.

Module 2: Automorphisms of a group, inner automorphisms, group of automorphisms and their computations (in particular for finite and infinite cyclic groups), characteristic subgroups, commutator subgroup and its properties.

Module 3: Direct product of groups, properties of external direct products, $\mathbb{Z}_n$ as external direct product, internal direct products, fundamental theorem of finite abelian groups, fundamental theorem of finitely generated abelian groups (statement only) and its applications.

Module 4: Rings: Polynomial rings over commutative rings, division algorithm and consequences, factorisation of polynomials, reducibility tests, irreducibility tests, Eisenstein's criterion. Some special ideals: Nilradical, Annihilating Ideal, Jacobson radical, Principal Ideal Domains (PID), unique factorisation in $\mathbb{Z}[x]$, divisibility in integral domains, irreducibles and primes, Unique Factorisation Domains (UFD), Euclidean Domains (ED), Local Rings.

Module 5: Applications of Algebra in various fields.

Books Recommended:

1. J. A. Gallian, Contemporary Abstract Algebra, Narosa.

2. J. B. Fraleigh, A First Course in Abstract Algebra, Pearson.
3. M. Artin, Abstract Algebra, Pearson.
4. T. W. Hungerford, Algebra, Springer.
5. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley.
6. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul, Basic Abstract Algebra, CUP.
7. M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.
8. J. J. Rotman, An Introduction to the Theory of Groups, Springer.
9. C. Musili, Introduction to Rings and Modules, Narosa.

Core 13: Complex Analysis and Fourier Series
Subject Code: MATH 06C13
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 conceptualize *differentiability* - *analyticity* - *holomorphicity* of a complex function.

CO 2 compute complex integrals.

CO 3 learn about singularities and apply techniques of meromorphic functions.

CO 4 apply the Fourier series expansion of periodic functions.

Detailed Syllabus

Complex Analysis:
Module 1: Complex numbers, geometric interpretation, stereographic projection, functions of complex variable, mappings, derivatives, holomorphic functions, differentiation formulas, Cauchy-Riemann equations, sufficient conditions for differentiability.

Module 2: Analytic functions and power series, analytic functions are holomorphic, absolute and uniform convergence of power series, examples of analytic functions, exponential function, trigonometric function, complex logarithm.

Module 3: Rectifiable paths, Riemann-Steieltjes integral of a function over an interval, complex integration of functions over a rectifiable path, contours and toy contours, contour integrals and its examples, upper bounds for moduli of contour integrals, Cauchy-Goursat theorem, Cauchy's integral formula, equivalence of analyticity and holomorphicity.

Module 4: Liouville's theorem and the fundamental theorem of algebra. Zeros of analytic functions and identity principle. Morera's theorem. Convergence of sequences and series of analytic functions.

Module 5: Poles, removable and essential singularity, Riemann's theorem on removable singularities, residue formula, Casorati-Weierstrass theorem, meromorphic functions. The argument principle, The open mapping property of holomorphic functions, maximum modulus principle, Schwarz lemma, conformality, Möbius transformations and the cross ratio, winding number, Laurent series.

Fourier series:
Module 6: Complex valued periodic functions on $\mathbb{R}$, inner products on periodic functions, trigonometric polynomials, Fourier series and coefficients, periodic convolutions, Weierstrass' approximation theorem for trigonometric polynomials, Fourier's theorem on mean square convergence, Bessel's inequality, Riemann-Lebesgue lemma, Parseval's identity, Dirichlet's theorem on convergence of Fourier series (proof can be done if time permits).

Books Recommended:

1. J. W. Brown and R. V. Churchill, Complex Variables and Applications, McGraw-Hill.
2. J. B. Conway, Functions of One Complex Variable, Springer.
3. E. M. Stein and R. Shakarchi, Complex Analysis, Princeton University Press.
4. T. W. Gamelin, Complex Analysis, Springer.
5. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill.
6. T. Tao, Analysis II, HBA.
7. T. M. Apostol, Mathematical Analysis, Narosa.

Core 14: Probability Theory
Subject Code: MATH 06C14
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 construct sample spaces associated with random experiments; demonstrate knowledge of random variables and their probability distributions, types of random variables with examples, functions of a random variable and their distributions;

CO 2 illustrate an understanding of a random vector and probability distribution of a random vector;

CO 3 demonstrate knowledge of various modes of convergence, weak and strong law of large numbers along with applications;

CO 4 demonstrate knowledge of the central limit theorem for independent and identically distributed random variables with examples.

Detailed Syllabus

Module 1: Classical theory and its limitations: Random experiment and events, event space; classical definition of probability and its drawback, statistical regularity, frequentist probability.

Module 2: Probability axioms: Basics from measure theory (sets, collections of sets, generators, the monotone class theorem, probability measure, probability space, the Borel sets on $\mathbb{R}$), continuity of probability, exclusion-inclusion formula, conditional probability and Bayes' rule, Boole's inequality, independence of events, Bernoulii trials and binomial law, Poisson trials, probability on finite sample spaces, geometric probability.

Module 3: Random variables and their probability distributions: Random variables, probability distribution of a random variable, discrete and continuous random variable, some discrete and continuous distributions on $\mathbb{R}$: Bernoulli, binomial and Poisson; uniform, normal, gamma, Cauchy and χ^2-distributions; functions of a random variable and their probability distributions.

Module 4: Moments and generating functions: Expectation, moments, measures of central tendency, measures of dispersion, measures of skewness and curtois, moment inequalities, probability generating function, moment generating function.

Module 5: Probability distributions on $\mathbb{R}^n$: Random vectors, probability distribution of a random vector, functions of random vectors and their probability distributions, moments, generating functions, correlation coefficient, conditional expectation, the principle of least squares, regression.

Module 6: Convergence: Sequence of random variables, almost sure convergence, convergence in probability, convergence in distribution, convergence in rth mean weak and strong law of large numbers, applications of the law of large numbers, Borel-Cantelli lemmas.

Module 7: Characteristic functions: Definition, properties, inversion, uniqueness, continuity theorem, Perseval's relation, Taylor expansion of characteristic functions.

Module 8: Central limit theorems: Lindeberg-Lévy central limit theorem, Lindeberg condition, Lyapunov condition, Lindeberg-Feller central limit theorem (statement only).

Books Recommended:

1. W. Feller, An introduction to probability theory and its applications I, J. Wiley.
2. G. Stirzaker and D. Grimmett, Probability and Random Processes, Oxford University Press.
3. V. K. Rohatgi and A. K. Md E. Saleh, An introduction to probability and statistics, John Wiley.
4. R. Durett, Probability Theory & Examples, Cambridge University Press.
5. Allan Gut, Probability-A Graduate Course, Springer.

Discipline Specific Elective Courses:

DSE 1: A. Linear Programming and Game Theory
Subject Code: MATH 05DSE1-A
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 gain insights in managerial decision to choose the best possible course of action to optimize resource allocation of a real-life problem keeping in mind the linear constraints involved: this has useful application in logistics and economical systems;

CO 2 understand the dual nature of real-life problems and how to utilise the duality to solve a given problem more easily;

CO 3 implement game theory in the study of mathematical models of strategic interaction between rational decision-makers and its applications in different fields of social sciences;

CO 4 solve unconstrained and constrained nonlinear and integer programming problems.

Detailed Syllabus

Module 1: General form of linear programming problems: basic formulation and geometric interpretation, standard and canonical forms. Basic solutions, examples, feasible solutions, degenerate solutions, reduction of a feasible solution to a basic feasible solution; convex set of feasible solutions of a system of linear equations and linear in-equations; extreme points, extreme directions, and boundary points of a convex set, describing convex polyhedral sets in terms of their extreme points and extreme directions: Caratheodory's representation theorem; correspondence between basic feasible solution of a system of linear equations and extreme point of the corresponding convex set of feasible solutions.

Module 2: Theory of simplex method, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, artificial variables: two-phase method, Big-M method and their comparison, special cases, Bland's rule, limitations of simplex method, Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual, introduction to interior point method, Karmarkar's methods.

Module 3: Transportation problem, mathematical formulation, north-west-corner method, least cost method, and Vogel approximation method for determination of starting basic solution, algorithm for solving transportation problem, special cases, assignment problem as special case of transportation problem, mathematical formulation, Hungarian method for solving assignment problem, special cases, travelling salesman problem.

Module 4: formulation of two person zero sum games, solving two person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

Module 5: Brief introduction to integer and non-linear programming.

Books Recommended:

1. G. Hadley, Linear Programming, Narosa.
2. H. Karloff, Linear Programming, Modern Birkhäuser Classic.
3. D. G. Luenberger and Y. Ye, Linear and nonlinear programming, Springer.
4. M. J. Osborne and A. Rubinstein, A Course in Game Theory, The MIT Press.
5. R. Myerson, Game Theory, Harvard University Press.
6. S. R. Chakravarty, M. Mitra and P. Sarkar, A Course in Cooperative Game Theory, Cambridge University Press.

DSE 1: B. Discrete Mathematics and Number Theory
Subject Code: MATH 05DSE1-B
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 efficiently handle discrete structures; solve concrete combinatorial problems in science, engineering and real life;

CO 2 use graphs as a unifying theme for various combinatorial problems;

CO 3 apply combinatorial intuitions in network theory, data structure and various other fields of science;

CO 4 apply number theory in real life problems, like cyber security.

Detailed Syllabus

Module 1: (Combinatorics) Basic counting principles: principle of inclusion exclusion, recurrence relations.

Module 2: (Graph Theory) Graphs and digraphs, complement, isomorphism, connectedness and reachability, adjacency matrix. Eulerian and Hamiltonian paths and circuits in graphs, Dijkstra's shortest path Algorithm, Trees, rooted trees and binary trees, Spanning Tree, Kruskal's algorithm, Prim's Algorithm, planar graphs, Euler's formula, statement of Kuratowski's theorem; independence number and clique number, chromatic number, statement of Havel-Hakimi Theorem and Four-color theorem.

Module 3: (Number Theory) Divisibility, primes and unique factorisation; GCD and Euclidean algorithm and its extension for computing multiplicative inverses; Arithmetic functions or number theoretic functions: sum and number of divisors, (totally) multiplicative functions, the greatest integer function, Euler's phi-function, Mobius function; definition and properties of the Dirichlet product; some properties of the Euler's phi-function, statement of the prime number theorem. Linear Diophantine equations, congruences and complete residue systems; quadratic residues, quadratic reciprocity and the law of quadratic reciprocity, Euler's criterion, Legendre symbol and Jacobi symbol, Euler-Fermat theorem, Wilson's theorem, Chinese remainder theorem.

Module 4: (Cryptography) Public-key encryption, Solovay–Strassen primality testing algorithm, notion of algorithms and their complexity, order notation, polynomial time algorithm, idea of hardness of factoring and discrete logarithm problem; basics of Diffie-Hellman key agreement and RSA encryption and decryption.

Module 5: (Practical) Implementation of number theoretic problems, implementation of Public Key Cryptosystems etc.

Books Recommended:

1. F. Roberts, Applied Combinatorics.
2. T. Andreescu and Z. Feng, A Path to Combinatorics for Undergraduates: Counting Strategies, Birkhauser.
3. D. B. West, Introduction to Graph Theory, PHI.
4. F. Harary, Graph Theory, Narosa.
5. D. M. Burton, Elementary Number Theory, TMH.
6. M. R. Adhikari and A. Adhikari, Basic Modern Algebra with Applications, Springer.
7. G. A. Jones and J. M. Jones, Elementary Number Theory, Springer.
8. N. Koblitz, A course in number theory and cryptography, Springer.
9. I. Niven, H. S. Zuckerman and H. L. Montgomery, An Introduction to the Theory of Numbers, John Wiley.

DSE 2: A. Theory of Ordinary Differential Equations
Subject Code: MATH 05DSE2-A
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the existence and uniqueness of solutions of the ordinary differential equations;

CO 2 investigate and classify the linear flows on $\mathbb{R}^n$ upto topological conjugacy and equivalency;

CO 3 characterize the qualitative nature of a flow in a neighborhood of a singularity;

CO 4 understand the local structure of limit sets.

Detailed Syllabus

Module 1: Fundamental theorem for existence and uniqueness, Gronwall's inequality, dependence on initial conditions and parameters, maximal interval of existence, global existence of solutions, vector fields and flows, topological conjugacy and equivalency.

Module 2: Linear flows on $\mathbb{R}^n$, The matrix exponential, linear first order autonomous systems, Jordan canonical forms, invariant subspaces, stability theory, classification of linear flows, fundamental matrix solution, non-homogeneous linear systems.

Module 3: Local structure of critical points (statements and applications of the local stable manifold theorem, the Hartman-Grobman theorem, the center manifold theorem), stability, Lyapunov function, gradient and Hamiltonian systems.

Module 4: α & ω limit sets of an orbit, attractors, periodic orbits, limit cycles and separatrix cycles, the Poincaré map, characterstic multipliers, Floquet Theory, the Poincaré-Benedixson theorem. Benedixson's criteria, Dulac's criteria, Liénard systems.

Books Recommended:

1. C. Chicone, Ordinary differential Equations with applications, Springer.
2. L. D. Perko, Differential Equations and Dynamical Systems, Springer.
3. E. A. Coddington and N. Levinson, Theory of Ordinary Differential Equations, Tata McGraw Hill-Publishing Co. Ltd.

DSE 2: B. Mathematical Modelling
Subject Code: MATH 05DSE2-B
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand and apply discrete and continuous models in physical and biological systems;

CO 2 apply sensitivity analysis to modeling problems;

CO 3 understand the autonomous system and apply the bifurcation analysis;

CO 4 solve real life modeling problems using programming softwares.

Detailed Syllabus

Module 1: Introduction: an overview

Module 2: Discrete models: Motivation, Examples, Solution and equilibrium of the discrete models, Cobwebbding method, General Theory and analytical methods.

Module 3: Continuous models: Motivation and derivation of continuous models, Differential Equation models, Separation of variables, Linear equations.

Module 4: Sensitivity Analysis: motivation and application.

Module 5: Systems of difference equations (discrete): Analytical methods and examples.

Module 6: Systems of Differential equations (Continuous): Motivations and some examples, Non-dimensionalization, analytical methods, Higher-order systems.

Module 7: Bifurcation analysis: Saddle-node, Transcritical, Pitchfork (both one and two dimensions), introduction of Hopf bifurcation, normal form of Hopf bifurcation.

Programming using any mathematical software

1. Plot for discrete models
2. Plot for continuous models
3. Plot for sensitivity
4. Plot for bifurcation
5. Plot for data fitting

Books Recommended

1. F. R. Giordano, M. D. Weir and W. P. Box, A first course in mathematical modelling, Thomson Learning.

2. L. E. Keshet, Mathematical models in biology, SIAM, 1988.

3. J. D. Murray, Mathematical Biology, Springer, 1993.

4. F. Brauer, P.V.D.Driessche, J. Wu, Mathematical Epidemiology, Springer, 2008.

DSE 3: A. Linear Algebra II and Field Theory
Subject Code: MATH 06DSE3-A
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the bilinear forms, alternating forms, and their classifications; understand the applications of spectral theory in finite dimensional case and applications of simultaneous diagonalization;

CO 2 determine the rational canonical form and Jordan form of linear transformations;

CO 3 understand the fundamental concepts of algebraic number fields, infinite fields of positive characteristic;

CO 4 understand the classification of finite fields.

Detailed Syllabus

Module 1: Linear Algebra: Billinear and Quadratic forms, positive and negative definite matrices; extrema of positive definite quadratic forms. Canonical forms: rational form and Jordan form of a matrix.

Module 2: Formulae of determinant and inverse of a partitioned matrix, idempotent matrices, left inverse and right inverse of full-rank rectangular matrices, generalized inverse.

Module 3: Proof of spectral theorem for complex Hermitian, normal and real symmetric matrices, singular value decomposition, polar decomposition, simultaneous diagonalization of commuting Hermitian/real symmetric matrices.

Module 4: Field theory: Examples: 1) field of fractions of an integral domain, 2) field of rational polynomials. 3) field of Meromorphic functions, 4) $R[x]/M$, where M is a maximal ideal of $R[x]$.

Module 5: Field extensions, finite and algebraic extensions, algebraic closure of a field, splitting fields; normal extensions, separable extensions, inseparable and purely inseparable extensions, simple extensions; solvability by radicals, radical extensions, ruler and compass constructions.

Module 6: Finite fields: structure of finite fields, existence and uniqueness theorems; Examples of construction of finite fields of order p^2, p^3 etc., primitive elements, minimal polynomials of elements, irreducible and primitive polynomials.

Books Recommended:

1. K. Hoffman and R. A. Kunze, Linear Algebra, PHI.

2. G. Strang, Linear Algebra and its Applications, Academic.
3. P. R. Halmos, Finite Dimensional Vector Spaces, Springer.
4. S. H. Friedberg, A. Insel and L. E. Spence, Linear Algebra, Pearson.
5. D. S. Dummit and R. M. Foote, Abstract Algebra, John Wiley.
6. T. W. Hungerford, Algebra, Springer.
7. P. Morandi, Field and Galois Theory, Springer.
8. J. Howie, Fields and Galois Theory, Springer (UMS).

DSE 3: B. A Mathematical Primer to Data Science
Subject Code: MATH 06DSE3-B
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 solve the system of linear equations using matrices, and compute Gram-Schmidt orthogonalizations, QR-decomposition, diagonalization of matrices; understand singular value decomposition (SVD) and its geometric intuition, SVD theorem and computational techniques, dimensionality reduction through principal component analysis;

CO 2 compute partial, directional and total derivatives of vector valued functions, gradients and Jacobians, useful gradient identities, and understand the geometric idea of gradient descent method;

CO 3 use various tools in probability and statistics, such as, properties of various univariate and multivariate probability distributions, estimators - MVUE, MLE etc., testing of hypotheses;

CO 4 apply various regression and classification techniques which are extremely important in machine learning and data science.

Detailed Syllabus

Module 1: (Linear Algebra) Quick Review: Solving system of linear equations using matrices, vector spaces, basis and dimension; linear transformations; inner product spaces and orthogonality; orthogonal complement and projections, orthonormal basis; Gram-Schmidt orthogonalization and QR-decomposition. Determinant and trace; eigenvalues and eigenvectors; eigen-decomposition(ED) and diagonalization.

Module 2: Singular value decomposition (SVD) and it's geometric intuition, SVD theorem, computation techniques for SVDs (through examples/practicals); ED vs. SVD. Elements of Dimensionality reduction through PCA.

Module 3: (Vector Calculus) Quick Review: Differentiability of vector valued functions: total derivative, directional derivatives, partial derivatives, gradient, Jacobian. Gradient of a Vector-Valued Function: Jacobian as a 'gradient'. Gradients of vector valued functions w.r.t. matrices, gradient of matrix valued functions w.r.t. matrices. Useful gradient identities.

Module 4: (Probability) Quick Review: Random variables and probality distributions, discrete and continuous distributions, Bayes' theorem. (Population and sample) Mean, variance, and covariance: univariate and multivariate, correlation.

Module 5: (Statistics) Statistical independence and inner products. Sampling based on normal, multivariate normal populations, t, χ^2 and F distributions, change of variables, sufficient statistics. Parameter estimation: consistency, minimum variance unbiased estimator (statement only), method of moments estimators, maximum likelihood estimator(MLE), consistency and asymptotic normality of MLE's (statement only). Testing of Hypothesis: one sample and two sample tests based on t, χ^2 and F distributions. Error probabilities, statistical power of test.

Module 6: (Regression and Classification) Linear regression: formulation and fitting straight lines using MLE, finding fitting/over-fitting errors: negative log-likelihood, root mean square error. Multiple linear regression. Bayesian linear regression. Principal component regression. Outlier detection. *Classification*: Linear classifiers, linear discriminant analysis (LDA), Quadratic discriminant analysis, logistic regression.

Books Recommended:

1. I. Miller and M. Miller, John Freund's Mathematical Statistics with Application, Pearson.
2. T. Hastie, R. Tibshirani and J. Friedman, The Elements of Statistical Learning, Springer.
3. M. P. Deisenroth, A. A. Faisal and C. S. Ong, Mathematics for Machine Learning, Cambridge University Press.
4. S. Shalev-Shwartz, S. Ben-David, Understanding Machine Learning: From Theory to Algorithms, Cambridge University Press.

DSE 4: A. Mechanics
Subject Code: MATH 06DSE4-A
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the concept of the moment of a force about a point or line, couple and their applications in problems;

CO 2 analyze the dynamics of a particle moving in one, two and three dimensions;

CO 3 understand the concept of a central force field and the dynamics of a particle under the central force field, and Kepler's laws of planetary motion;

CO 4 understand the concepts of constrained motion and derive the Lagrange's equations of motion with the use of D'Alemdert's principle and the principle of least action;

Detailed Syllabus

Module 1: (Statics) Coplanar forces, astatic equilibrium, friction, principle of virtual work, stable and unstable equilibrium, centre of gravity for different bodies, general conditions of equilibrium, forces in three dimensions.

Module 2: (Particle dynamics) Newton's equation of motion of a particle, simple illustrations: simple harmonic motion, particle in a central force field; central orbits and Kepler's laws, Simple harmonic Motion.

Module 3: (Rigid-body dynamics) Moments and products of inertia, D'Alembert's principle of motion, compound pendulum, motion in two dimensions, conservation of linear and angular momentum, conservation of energy; derivation of Lagrange's equations of motion for conservative holonomic system from D'Alembert's principle and from variational principle; solution of problems by Lagrange's equation.

Module 4: Visualization of some dynamical problems using any mathematical software.

Books Recommended:

1. S. L. Loney, Elements of Statics and Dynamics 1 and 2, Arihant Publications.
2. S. L. Loney, An Elementary treatise on Dynamics of particle and rigid bodies, New Age International Private Limited.
3. F. Chorlton, Textbook of Dynamics, John Wiley & Sons.
4. N. C. Rana and P. S. Joag, Classiscal Mechanics, McGraw-Hill.
5. H. Goldstein, Classical Mechanics, Pearson.

DSE 4: B. Differential Geometry
Subject Code: MATH 06DSE4-B
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand the basic concepts of smooth curves in three dimensions such as curvature, torsion, Frenet formulas;

CO 2 know the geometry of regular surfaces in three dimensions and hypersurfaces in $(n + 1)$ dimensions;

CO 3 understand first and second fundamental forms of regular surfaces, Gauss map, Gaussian curvature of regular surfaces;

CO 4 know covariant derivatives, geodesics, exponential maps of regular surfaces;

Detailed Syllabus

Module 1: Review of multivariate calculus: directional and total derivatives, inverse & implicit function theorems.

Module 2: Theory of curves: definition and examples of parametrized curves in $\mathbb{R}^3$, regular curves, arc lengths, reparametrization of regular curves, curvature, torsion, Frenet formulas, fundamental theorem of the local theory of curves, Isoperimetric inequality and the four vertex theorem for plane curves.

Module 3: Theory of surfaces: definition and examples of regular surfaces in $\mathbb{R}^3$, smooth functions on a regular surface and smooth maps between two regular surfaces, tangent plane to a regular surface at a point, differential of a smooth map, smooth vector field on a regular surface, integral curves and local flow of a smooth vector field.

Module 4: First fundamental form and its application: first fundamental form, arc length of a smooth curve in a regular surface, angle between two curves, area of a bounded region in a regular surface, conformal maps and isometries.

Module 5: Second fundamental form and its application: Orientation of a regular surface, Gauss map and its properties, second fundamental form and its geometric interpretation, Hessian of a smooth function as a second fundamental form, principal curvatures, Gaussian curvature and mean curvature, Dupin indicatrix, asymptotic directions, asymptotic curves, line of curvatures.

Module 6: Covariant derivatives and geodesics: covariant derivatives, Christoffel symbols, parallel transport, geodesics, the exponential map, geodesic coordinates.

Module 7: Differential forms: Differential forms in $\mathbb{R}^3$, exterior product and exterior derivative of forms, closed and exact forms, Poincaré lemma. Forms on surfaces, integration on surfaces, Stoke's theorem (statement only).

Books Recommended

1. J. A. Thorpe, Elementary Topics in Differential Geometry, Springer.
2. M. P. do Carmo, Differential Geometry of Curves and Surfaces, Dover.
3. A. Pressley, Elementary Differential Geometry, Springer (UMS).
4. M. P. do Carmo: Differential Forms and Applications, Springer (Universitext).

Skill Enhancement Courses:

SEC 1A: Computer Programming with C
Subject Code: MATH 03SEC1A
Credits: 4 (2 Theory lectures + 2 Practicals per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 be acquainted with the basics of programming language in connection with mathematics;

CO 2 apply loops and decision statements, arrays and pointers, dynamic memory allocation, functions and pass arguments in C programming;

CO 3 apply and use library functions, read and write files in C programming; in C programming;

CO 4 implement in many real life applications.

Detailed Syllabus

Module 1: Introduction to programming in the C language: variables, conditional statements, loops, arrays, functions, recursive programming.

Module 2: Pointer, dynamic memory allocation, linked lists; lists, stacks, queues and trees; File handling.

Module 3: Practicals: searching and sorting algorithms; programs related to number theory, numerical analysis, abstract algebra, geometry etc.

Books Recommended

1. B. W. Kernighan and D. M. Ritchie, The C programming Language, PHI.
2. E. Horowitz and S. Sahani, Fundamentals of Data Structure, Computer Science Press.

SEC 1B: Computer Programming with Python
Subject Code: MATH 03SEC1B
Credits: 4 (2 Theory lectures + 2 Practicals per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 be acquainted with the basics of programming language in connection with mathematics;

CO 2 learn how to use lists, tuples, and dictionaries, object types, indexing and slicing to access data in Python programs;

CO 3 learn how to write loops and decision statements, functions and pass arguments in Python;

CO 4 learn how to use library functions, read and write files in Python; implement a few real life applications.

Detailed Syllabus

Module 1: Introduction to Python: Downloading and installing Python. Understanding, how to use Python and PIP(Package Installer for Python). Installing and using IPython with Jupyter Notebook. (One can use Kaggle or Google Colab)
Module 2: Built In Data-Types: int, float, complex str, bool, set, dict; Iterators: list, range, str; Control Flow: Sequential, Decision(if-else, nested if-else), Repetition (for-loop, while-loop); Function: Function definition, Parameters, Arguments, Local variables, Calling a Function, Built-In Python Functions (abs(), any(), bin(), bytes(), chr(), com(), float(), format(), input(), int(), len(), list(), max(), min(), open(), pow(), print(), str(), sum() etc.).
Module 3: Python Strings: Replace, Join, Split, Reverse, Uppercase, Lowercase, etc. Use of Len(), index(), find(), join() etc.
Module 4: Packages and Modules: Numpy, Matplotlib, Pandas etc.
Module 5: Practicals: searching and sorting algorithms; programs related to number theory, numerical analysis, abstract algebra, geometry, data science etc.

Books Recommended

1. V. L. Ceder, The Quick Python Book, Second Edition, Manning.
2. J. C. Bautista, Mathematics and Python Programming, Lulu.com.
3. A. Saha, Doing Math with Python, No Starch Press.
4. C. Satyanarayana, M. R. Mani and B. N. Jagadesh, Python Programming, University Press.

SEC 2: Latex
Subject Code: MATH 04SEC2
Credits: 4 (2 Theory lectures + 2 Practicals per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 understand how latex brings ease in mathematical writing;

CO 2 write projects and papers with latex;

CO 3 insert tables, figures, equations in a document; create bibliographies;

CO 4 make easily customizable presentations.

Detailed Syllabus

Motivation behind learning Latex; Document structure; Typesetting text, Math Mods; Tables; Figures; Equations; Referencing; Beamer presentation.

Books Recommended:

1. H. Kopka and P. W. Daly, Guide to Latex, Addison-Wesley.

2. S. Kottwitz, Latex Beginner's Guide, Packt Publishing Ltd.

General Elective Courses (to be offered to the students of other departments):

GE 1: Differential Calculus
Subject Code: MATH 01GE1
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 acquaint with the real numbers and learn the construction of number systems;

CO 2 understand the concept of sequence and series of real numbers and their convergences; understand the limit and continuity of a function at a point;

CO 3 familiarize with the important properties of the continuous functions on closed intervals; acquire knowledge of derivatives of functions at points with various examples and its geometrical and physical interpretation;

CO 4 understand the important results of differentiable functions on intervals and their applications to differential calculus, e.g. maxima-minima, tangent, normals, etc; understand indeterminate forms and use of L'Hôpital's rule.

Detailed Syllabus

Module 1: Real Numbers: Axiomatic definition, Archimedean property, limit supremum, limit infimum.

Module 2: Sequence of real numbers: convergence, Cauchy criteria and other elementary properties. Series of real number, Absolute and conditional convergence of series.

Module 3: Real-valued functions defined on an interval : Limit of a function (Cauchy's definition). Algebra of limits. Continuity of a function at a point and in an interval. Acquaintance with the important properties of continuous functions on closed intervals.

Module 4: Derivative its geometrical and physical interpretation. Sign of derivative, Monotonic increasing and decreasing functions. Relation between continuity and differentiability.

Module 5: Successive derivative (Leibnitz's Theorem and its application).

Module 6: Rolle's theorem; Mean Value Theorems and expansion of functions like e^x; $\sin x$; $\cos x$; $(1+x)^n$; $\ln(1+x)$ (with validity of regions).

Module 7: Applications of Differential Calculus : Maxima and Minima, Tangents and Normals.

Module 8: Indeterminate Forms: L'Hôpital's Rule.

Books Recommended:

1. R. G. Bartle and D. R. Sherbert, Introduction to Real Analysis, John Wiley & Sons, Inc.
2. T. M. Apostol, Calculus (Vol. I), Wiley.
3. D. V. Widder, Advanced Calculus, Dover Publications.
4. S. Narayan, Differential Calculus, S. Chand.

GE 2: Integral Calculus and Differential equations
Subject Code: MATH 02GE2
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

After successful completion of this course, a student will be able to:

CO 1 integrate rational functions and apply reduction formulas of integrations; do definite integrals as the area under a curve;

CO 2 understand the concept of Riemann integral to functions defined on unbounded intervals or to unbounded functions i.e. the concept of improper Riemann integrals, related tests and Beta and Gamma functions;

CO 3 apply integral calculus to find the length of a curve, quadrature, volume and surface areas of solids formed by revolution of plane curve and areas, etc;

CO 4 apply various techniques for solving first and higher order linear differential equations.

Detailed Syllabus

Module 1: Integration of the form $\int \frac{dx}{a + b\cos x}$, $\int \frac{l\sin x + p\cos x}{m\sin x + n\cos x}dx$ and integration of rational functions. Reduction formulae of $\int \sin^m x\cos^n x dx$; $\int \tan^n x dx$ and $\int \frac{\sin^m x}{\cos^n x}dx$ and associated problems (m and n are non-negative integers).

Module 2: Evaluation of definite integrals. Preliminaries of Riemann integration. Integration as the limit of a sum.

Module 3: Definition of Improper Integrals: Statements of (i) μ-test, (ii) Comparison test. Use of Beta and Gamma functions.

Module 4: (Applications of integral calculus) rectification, quadrature, finding c.g. of regular objects, volume and surface areas of solids formed by revolution of plane curve and areas.

Module 5: Introduction to differential equations; Order and solution of an ordinary differential equation (ODE) in presence of arbitrary constants; Formation of ODE.

Module 6: First order differential equations: (i) Variables separable, (ii) Homogeneous equations and equations reducible to homogeneous forms, (iii) Exact equations and those reducible to such equation, (iv) Euler's and Bernoulli's equations (Linear), (v) Clairaut's Equations: General and Singular solutions; Orthogonal Trajectories.

Module 7: Second order linear equations: Second order linear differential equations with constant coefficients. Euler's Homogeneous equations.

Books Recommended

1. S. Narayan, Integral Calculus, S. Chand.
2. T. M. Apostol, Calculus (Vol. I), Wiley.
3. S. L. Ross, Differential Equations, John Wiley and Sons.
4. G. F. Simmons, Differential Equation with Applications and Historical Notes, CRC Press.

GE 3: Algebra I
Subject Code: MATH 03GE3
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

This course will enable the students to:

CO 1 use De Moivre's theorem in a number of applications to solve mathematical problems;

CO 2 determine roots of real and complex polynomials using various methods;

CO 3 get a preliminary idea about groups, rings and fields.

Detailed Syllabus

Module 1: (Complex Numbers) De Moivre's Theorem and its applications. Exponential, Sine, Cosine and Logarithm of a complex number. Definition of e^z, Inverse circular and Hyperbolic functions.

Module 2: (Theory of Equations) Fundamental Theorem of Algebra. Polynomials with real coefficients: Descarte's Rule of sign and its applications. Relation between roots and coefficients. Symmetric functions of roots, Transformations of equations. Solution of a cubic and biquadratic.

Module 3: (Introduction to Group Theory) Definition and examples, Cyclic group, Symmetric group, Alternating group. Elementary properties of groups. Order of an element in the group, Subgroup, Quotient group, Normal subgroup, Homomorphism and isomorphism.

Module 4: (Rings and Integral Domains) Definition and examples. Subrings and ideals. Quotient ring. Homomorphism ans isomorphism of rings.

Module 5: (Fields) Definition and examples, its relation with integral domain.

Books Recommended

1. S. K. Mapa, Classical Algebra, Levant.

2. J. B. Fraleigh, First Course in Abstract Algebra, Narosa.

3. M. K. Sen, S. Ghosh and P. Mukhopadhyay, Topics in Abstract Algebra, University Press.

GE 4: Algebra II
Subject Code: MATH 04GE4
Credits: 6 (5 Theory lectures + 1 Tutorial per week)

Outcomes of the Course

This course will enable the students to:

CO 1 have basic understanding of vector spaces;

CO 2 recognize consistent and inconsistent systems of linear equations by the row echelon form of the augmented matrix, using rank; and solve consistent systems of linear equations;

CO 3 have basic knowledge of linear transformations;

CO 4 find eigenvalues and corresponding eigenvectors for a square matrix.

Detailed Syllabus

Module 1: Vector (Linear) space over a field. Subspaces. Linear combinations. Linear dependence and independence of a set of vectors. Linear span. Basis. Dimension. Replacement Theorem. Extension theorem. Deletion theorem.

Module 2: Row Space and Column Space of a Matrix. Determinant and Trace of a matrix. Rank of a matrix. $\text{Rank}(AB) \leq \min(\text{Rank } A; \text{Rank } B)$.

Module 3: System of Linear homogeneous equations: Solution space of a homogeneous system and its dimension. System of linear non-homogeneous equations: Necessary and sufficient condition for the consistency of the system. Method of solution of the system of equations.

Module 4: Linear Transformation (L.T.) on Vector Spaces: Null space. Range space. Rank and Nullity, Sylvester's law of Nullity. Inverse of Linear Transformation. Non-singular Linear Transformation. Change of basis by Linear Transformation. Vector spaces of Linear Transformation.

Module 5: Characteristic equation of a square matrix. Eigen-value and Eigen-vector. Invariant subspace. Cayley-Hamilton Theorem. Simple properties of Eigen value and Eigen vector, diagonalization.

Books Recommended

1. S. Kumaresan, Linear Algebra: A Geometric Approach, PHI.
2. B. Rao, Linear Algebra, HBA (TRIM).
3. S. H. Friedberg, A. Insel and L. E. Spence, Linear Algebra, Pearson.